Appendix G

Computing a Wage Trend Line

A wage trend line passes through an array of wage data so that the data above the line and the data below the line are in balance.

A mathematical equation for a trend line allows the trend of a series of data to be shown, and also provides a concise definition of that trend. Such an equation is used in the "least squares" method, accepted by statisticians as a sound, convenient device for obtaining an objective fit of a trend line.

In fitting a trend line to wage survey findings, the first trend lines used are straight lines fitted to the data by the "least squares" method.

Consistent with the name "least squares", the sum of the squares of the deviations of all the data, both above and below the fitted line, is less than the sum of the squares of the deviations of the data from any *other* straight line. Another characteristic of this is that the sum of the deviations of the data above the line exactly balances the sum of the deviations below.

In computing the trend line two methods are used:

(1) the so-called unit line approach which gives equal weight to each survey job weighted average and

(2) the so-called frequency line which weights each survey job average by the number of employees matched to the job in the survey.

The Formula for a straight line Y = a + bx

Y = computed value of trend

a = the value of the line at its origin

b = intergrade differential

x = any grade level

In order to compute a *unit* trend line under the "least squares" formula the values of the unknown "a" and "b" are derived through use of the following equations:

$$a = \frac{(\sum x^2)(\sum y) - (\sum x)(\sum xy)}{(N)(\sum x^2) - (\sum x)^2}$$

By substituting the appropriate column totals from columns (1) through (5) from the sample computation sheet we get the following:

$$a = \frac{(1587)(208.396) - (185)(1581.005)}{(26)(1587) - (185)^2}$$

$$a = \frac{330{,}724.45 - 292{,}485.92}{41{,}262 - 34{,}225}$$

$$a = \frac{38{,}238.53}{7{,}037}$$

$$a = 5.434$$

$$b = \frac{(N)(\sum xy) - (\sum x)(\sum y)}{(N)(\sum x^2) - (\sum x)^2}$$

(This divisor is identical to that used in the arithmetic computation shown for solving the unknown value of "a" above.)

$$b = \frac{(26)(1581.005) - (185)(208.396)}{(26)(1587) - (185)^2}$$

$$b = \frac{41{,}106.13 - 38{,}553.26}{41{,}262 - 34{,}225}$$

$$b = \frac{2{,}552.87}{7{,}037}$$

$$b = 0.363$$

To determine the wage rate for any given grade level the values for "a" and "b" are substituted in the original formula of Y = a + bx. Inserted in the place of x is the grade to be computed. For example:

Grade 1:

$$Y = a + bx = 5.434 + (.363)(1) = 5.797\ (5.80)$$

Grade 5:

$$Y = a + bx = 5.434 + (.363)(5) = 7.249\ (7.25)$$

Grade 10:

$$Y = a + bx = 5.434 + (.363)(10) = 9.064\ (9.06)$$

In computing a *frequency* trend line under the "least squares" formula the following equations are used:

$$a = \frac{(\sum fx^2)(\sum fy) - (\sum fx)(\sum fxy)}{(\sum f)(\sum fx^2) - (\sum fx)^2}$$

$$b = \frac{(\sum f)(\sum fxy) - (\sum fx)(\sum fy)}{(\sum f)(\sum fx^2) - (\sum fx)^2}$$

The equations are solved by substituting the appropriate column totals from columns (6) through (10) as shown in the sample computation sheet. To determine the wage rate for any given level the same procedures used for the unit line are followed.

Exhibit I. Computation Sheet

Unit line					*Frequency line*				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Example Survey Job	*Survey Job Grade*	*Weighted Average*	*(2) x (3)*	*(2)* ^{2} *(squared)*	*Number Survey Job Matches*	*(6) x (2)*	*(6) x (3)*	*(6) x (4)*	*(6) x (5)*
n	*x*	*y*	*xy*	x5	*f*	*fx*	*fy*	*fxy*	*fx5*
A	1	5.296	5.296	1	1948	1948	10316.605	10316.07	1948
B	2	6.529	13.058	4	2150	4300	14037.348	28074.701	8600
C	2	6.698	13.396	4	3865	7730	25887.770	51775.540	15460
D	3	7.762	23.286	9	1737	5211	13482.594	40447.774	15633
E	4	5.116	20.464	16	2078	8312	10631.047	42524.170	33248
F	4	6.209	24.836	16	125	500	776.125	3104.500	2000
G	5	6.801	34.005	25	66	330	448.866	2244.329	1650
H	5	6.501	32.505	25	2054	10270	13353.055	66765.249	51350
I	5	7.269	36.345	25	2757	13785	20040.633	100203.126	68925
J	5	7.918	39.590	25	439	2195	3476.002	17380.008	10975
K	6	8.004	48.024	36	1228	7368	9828.910	58973.456	44208
L	6	7.793	46.758	36	363	2178	2828.859	16973.152	13068
M	7	9.233	64.631	49	1884	13188	17394.969	121764.798	92316
N	8	8.279	66.232	64	5327	42616	44102.234	352817.835	340928
O	9	8.203	73.827	81	3821	34389	31343.660	282092.951	309501
P	9	8.167	73.503	81	137	1233	1118.879	10069.910	11097
Q	10	9.631	96.310	100	1227	12270	11817.234	118172.348	122700
R	10	9.212	92.120	100	846	8460	7793.352	77933.516	84600
S	10	9.080	90.800	100	224	2240	2033.920	20339.197	22400
T	10	8.080	80.800	100	1274	12740	10293.918	102939.184	127400
U	10	8.551	85.510	100	529	5290	4523.477	45234.787	52900
V	10	8.771	87.710	100	1536	15360	13472.254	134722.547	153600
W	10	10.448	104.480	100	205	2050	2141.840	21418.399	20500
X	10	9.638	96.380	100	2586	25860	24923.863	249238.653	258600
Y	11	9.276	102.036	121	222	2442	2059.272	22651.991	26862
Z	13	9.931	129.103	169	1330	17290	13208.227	171706.986	224770
N=26	∑x=185	∑y=280.396	∑xy= 1581.005	∑x5 =1587	∑f=39958	∑fx=259555	∑fy = 311334.911	∑fxy = 2169885.715	∑fx5 = 2115239

N = total number of survey jobs
x = grade
y = weighted average
f = number of survey job matches
∑ = summation