Comparision of "Old" and "New" Concepts: Coordinate Times and Time Transformations

Gerard Petit

Bureau International des Poids et Mesures,
Pavillon de Breteuil 92312 Sevres Cedex France,
e-mail: gpetit@bipm.org

Foreword: The first three parts of this paper are largely based on the paper "The new IAU'2000 Conventions for coordinate times and time transformations" presented at the Journées Systèmes de Référence Spatio-temporels, Brussels, September 2001. The fourth part is largely based on draft parts of the IERS Conventions 2000 (`http://maia.usno.navy.mil/conv2000.html`), interpreted here under the sole responsibility of the author.

Abstract: At its 2000 General Assembly, the International Astronomical Union has adopted a set of Resolutions that provide a consistent framework for defining the barycentric and geocentric celestial reference systems at the first post-Newtonian level. This extends and completes the IAU'1991 framework defined in Resolution A4 at the 1991 General Assembly. This paper describes in some detail the two Resolutions that define time coordinates and allow to realize time transformations. The application of the IAU'1991 and IAU'2000 framework to several fields of space geodesy and astrometry is discussed.

1 Introduction

At its 1991 General Assembly the International Astronomical Union (IAU) explicitly adopted the general theory of relativity as the theoretical framework for the definition and realization of space-time reference frames (IAU, 1991). Barycentric and geocentric coordinate time scales and the relativistic transformations between them were defined, together with procedures for their realization. In section 2, we recall the content of the IAU 1991 resolution A4 dealing with the definition of reference systems, time coordinates and time transformations, and we expose some of the limitations of this framework. For many applications in space geodesy, it is sufficient to discuss within the IAU'1991 framework, with the IAU'2000 framework providing an extension which is necessary for a few applications. It is therefore important to have in mind the basis of the IAU'1991 framework.

The IAU Working Group on Relativity in Celestial Mechanics and Astrometry (RCMA), since 1994, and the BIPM/IAU Joint Committee on relativity for space-time reference systems and metrology (JCR), from 1997 to 2001, have worked to provide an extension of the IAU'1991 framework at the first post-Newtonian level (Soffel, 2000; Petit, 2000). This work resulted in a set of Resolutions passed at the IAU 2000 General Assembly. The complete text of the Resolutions may be found in IAU publications (`http://danof.obspm.fr/IAU-resolutions/Resol-UAI.htm`) and a complete explanatory supplement may be found in (Soffel et al., 2002). The new IAU'2000 framework is precisely specified in Resolutions B1.3(2000) "Definition of barycen-

tric celestial reference system and geocentric celestial reference system" and B1.4(2000) "Post-Newtonian potential coefficients".

In Section 3 of this paper, we briefly present the two Resolutions which deal with time transformations and the definition of coordinate times, namely Resolutions B1.5(2000) "Extended relativistic framework for time transformations and realization of coordinate times in the solar system" in section 3.1 and B1.9(2000) "Re-definition of Terrestrial Time TT" in section 3.2, and discuss at which level time transformations and coordinate times are affected by the IAU'2000 framework.

Finally section 4 discusses the application of the IAU'1991 and IAU'2000 framework to several fields of space geodesy and astrometry which are based on time and frequency measurements.

2   The IAU'1991 framework and its limitations

The IAU resolution A4 (1991) contains nine recommendations, the first five of which are directly relevant to our discussion.

In the first recommendation, the metric tensor for space-time coordinate systems $(t, \mathbf{x})$ centered at the barycenter of an ensemble of masses is recommended in the form

$$
\begin{aligned}
g_{00} &= -1 + \frac{2U(t, \mathbf{x})}{c^2} + \mathcal{O}(c^{-4}), \\
g_{0i} &= \mathcal{O}(c^{-3}), \\
g_{ij} &= \delta_{ij} \left(\frac{1 + 2U(t, \mathbf{x})}{c^2} \right) + \mathcal{O}(c^{-4}).
\end{aligned}
\tag{1}
$$

where c is the speed of light in vacuum ($c = 299792458$ m/s), U is the sum of the gravitational potentials of the ensemble of masses and of a tidal potential generated by bodies external to the ensemble, the latter potential vanishing at the barycenter. The algebraic sign of U is taken to be positive. This recommendation recognizes that space-time cannot be described by a single coordinate system. The recommended form of the metric tensor can be used not only to describe the barycentric reference system (BRS) of the whole solar system (which is called BCRS where C stands for Celestial since the IAU'2000 Resolutions), but also to define the geocentric reference system (GRS) centered in the center of mass of the Earth, which is now called GCRS. In analogy to the GRS, a corresponding reference system may be defined for any other body of the Solar system.

In the second recommendation, the origin and orientation of the space coordinate grids for the solar system (BRS) and for the Earth (GRS) are defined. Notably it is specified that the space coordinate grids of these systems should show no global rotation with respect to a set of distant extragalactic objects. It also specifies that the SI (International System of units) second and the SI meter should be the physical units of proper time and proper length in all coordinate systems. It states in addition that the time coordinates should be derived from an Earth atomic time scale.

The third recommendation defines TCB (Barycentric Coordinate Time) and TCG (Geocentric Coordinate Time) – the time coordinates of the BRS and GRS, respectively. The recommendation also defines the origin of the times scales (their reading on 1977 January 1, $0^{\mathrm{h}} 0^{\mathrm{m}} 0^{\mathrm{s}}$ TAI ($JD = 2443144.5\,TAI$) must be 1977 January 1, $0^{\mathrm{h}} 0^{\mathrm{m}} 32.184^{\mathrm{s}}$) and declares that the units of measurements of the coordinate times of all reference systems must be coincide with

the SI second and SI meter. The relationship between TCB and TCG is given by a full 4-dimensional transformation

$$TCB - TCG = c^{-2}\left[\int_{t_0}^{t}\left(\frac{v_E^2}{2} + U_{ext}(t, \mathbf{x}_E(t))\right)dt + v_E^i r_E^i\right] + \mathcal{O}(c^{-4}), \quad (2)$$

where x_E^i and v_E^i are the barycentric coordinate position and velocity of the geocenter, $r_E^i = x^i - x_E^i$ with x^i the barycentric position of the observer, and $U_{ext}(t, \mathbf{x}_E(t))$ is the Newtonian potential of all solar system bodies apart from the Earth evaluated at the geocenter.

In the fourth recommendation another time coordinate, Terrestrial Time (TT), is defined for the GRS. It differs from TCG by a constant rate only

$$TCG - TT = L_G \times (JD - 2443144.5) \times 86400, \qquad L_G \approx 6.969291 \times 10^{-10}, \quad (3)$$

so that the unit of measurement of TT agrees with the SI second on the geoid. TT represents an ideal form of TAI, the divergence between them being a consequence of the physical defects of atomic clocks.

The fifth recommendation states that the former dynamical barycentric time TDB may still be used where discontinuity with previous work is deemed to be undesirable.

Limitations to the IAU'1991 framework

Because of the form of the metric (1) in the IAU'1991 framework, time transformations and the realization of coordinate times in the barycentric system are not specified at the c^{-4} level, i.e. at a level of a few parts in 10^{16} in rate. The new IAU'2000 framework allows to remove this limitation. Nevertheless, within the IAU'1991 approximation, constants L_B and L_C were introduced in notes to the Recommendation 3 (1991) to express the mean rates between time scales as

$$
\begin{aligned}
TCB - TDB &= L_B \times (JD - 2443144.5) \times 86400, \\
&\quad L_B \approx 1.550505 \times 10^{-8}, \text{ and} \\
TCB - TCG &= L_C \times (JD - 2443144.5) \times 86400 + v_E^i r_E^i/c^2 + P, \\
&\quad L_C \approx 1.480813 \times 10^{-8}, \quad\quad\quad\quad (4)
\end{aligned}
$$

where P represents periodic terms. Since JD is not specified to be a particular time scale, these constants were not properly defined so that confusion appeared in their usage. This point was adressed in Resolution B1.5(2000) by defining $< TCG/TCB >= 1 - L_C$ and $< TT/TCB >= 1 - L_B$, where $<>$ means a sufficiently long term average taken at the geocenter. The actual computation of L_C and L_B requires the integration of solar system ephemerides and the specification of an averaging duration, and this process may be applied to the utmost accuracy, after a choice of ephemerides and averaging duration. For example Irwin and Fukushima (1999) determined $L_C = 1.48082686741 \times 10^{-8} \pm 2 \times 10^{-17}$. However, no completely unambiguous definition may be provided for L_B and L_C because they always depend on the ephemerides and time span used for their computation. Therefore the use of these constants is not advised to formulate time transformations when it would require knowing their value with an uncertainty of order 1×10^{-16} or less.

Another problem arising from the situation before IAU'1991 is that the barycentric dynamical time TDB did not have a good definition. This could have been corrected by turning a specific value of L_B into a defining constant thus providing in retrospect a definition of TDB. It was not felt necessary to address explicitly this point in a recommendation. Prior to IAU'1991, authors

had developed analytical formulas to transform TT (known as TDT prior to 1991) to TDB (e.g. Fairhead and Bretagnon, 1990). Such formulas may be used along with a further linear transformation to obtain TCB from TDB, i.e. $dTDB/dTCB = 1 - L_B$, under the same provisions as above for possible ambiguities in the definitions, and therefore in the applicability of the formulas. See more details in section 4.5 and see (Irwin and Fukushima, 1999) for a comparison of some transformation formulas and the level of uncertainty that may be associated with them.

3   Time Coordinates in the IAU'2000 framework

For practical applications concerning time and frequency measurements in the solar system, it is necessary to consider a conventional model for the realization of time coordinates and time transformations. This model should be chosen so that i) its accuracy is significantly better than the expected performance of clocks and time transfer techniques, ii) it is consistent with the general framework of IAU Resolutions B1.3 and B1.4 (2000) and may readily be used with existing astrometric quantities, e.g. solar systems ephemeris.

For the first condition, it was determined that time coordinates and time transformations should be realized with an uncertainty not larger than 5×10^{-18} in rate or, for quasi-periodic terms, not larger than 5×10^{-18} in rate amplitude and 0.2 ps in phase amplitude. For the spatial domain of validity, the formulations in the barycentic system are valid up to a few solar radii from the Sun and, in the geocentric system, locations from the Earth's surface up to geostationary orbits ($|\mathbf{X}| < 50000$ km) have been considered.

3.1   Resolution B1.5(2000): Extended relativistic framework for time transformations and realization of coordinate times in the solar system

Following Resolutions B1.3(2000) and B1.4(2000), the metric tensor in the BCRS is expressed as

$$g_{00} = -(1 - \tfrac{2}{c^2}(w_0(t,\mathbf{x}) + w_L(t,\mathbf{x})) + \tfrac{2}{c^4}(w_0^2(t,\mathbf{x}) + \Delta(t,\mathbf{x})))$$

$$g_{0i} = -\tfrac{4}{c^3}w_i(t,\mathbf{x})$$

$$g_{ij} = \left(1 + \tfrac{2w_0(t,\mathbf{x})}{c^2}\right)\delta_{ij}$$

(5)

where $(t \equiv TCB, \mathbf{x})$ are the barycentric coordinates, $w_0 = G\sum_A M_A/r_A$, with the summation carried out over all solar system bodies A, $\mathbf{r}_A = \mathbf{x} - \mathbf{x}_A$, $r_A = |\mathbf{r}_A|$, and where w_L contains the expansion in terms of multipole moments, as defined in Resolution B1.4(2000) and references therein, required for each body. In many cases the mass-monopole approximation ($w_L = 0$) may be sufficient to reach the above mentioned uncertainties but this term should be kept to ensure the consistency in all cases. The values of masses and multipole moments to be used may be found in IAU or IERS documents (IERS, 1996), but care must be taken that the values are in SI units (not in so-called TDB units or TT units). The vector potential $w^i(t,\mathbf{x}) = \sum_A w_A^i(t,\mathbf{x})$ and the function $\Delta(t,\mathbf{x}) = \sum_A \Delta_A(t,\mathbf{x})$ are given in the text of IAU Resolutions.

From (5) the transformation between proper time and TCB may be derived.

It reads:

$$\mathrm{d}\tau/\mathrm{d}TCB = 1 - \frac{1}{c^2}\left(w_0 + w_L + \frac{v^2}{2}\right)$$

$$+\frac{1}{c^4}\left(-\frac{1}{8}v^4 - \frac{3}{2}v^2 w_0 + 4v^i w^i + \frac{1}{2}w_0^2 + \Delta\right) \tag{6}$$

Evaluation of the significance of the terms in $w^i(t, \mathbf{x})$ and $\Delta(t, \mathbf{x})$ may be found in (Petit, 2001).

Similarly, the transformation between TCB and TCG may be written as

$$TCB - TCG = c^{-2}\left[\int_{t_0}^{t}\left(\frac{v_E^2}{2} + w_{0ext}(\mathbf{x}_E)\right)dt + v_E^i r_E^i\right]$$

$$-c^{-4}[\int_{t_0}^{t}\left(-\frac{1}{8}v_E^4 - \frac{3}{2}v_E^2 w_{0ext}(\mathbf{x}_E) + 4v_E^i w_{ext}^i(\mathbf{x}_E) + \frac{1}{2}w_{0ext}^2(\mathbf{x}_E)\right)dt$$

$$-(3w_{0ext}(\mathbf{x}_E) + v_E^2/2)v_E^i r_E^i] \tag{7}$$

where t is TCB and where the index ext refers to all bodies except the Earth. This equation is composed of terms evaluated at the geocenter (the two integrals) and of position dependent terms in r_E, with position dependent terms in higher powers of r_E having been found to be negligible. The first integral may be computed from existing planetary ephemeris (Fukushima, 1995; Irwin and Fukushima, 1999). Since, in general, the planetary ephemeris are expressed in terms of a time argument T_{eph} which is close to TDB (see section 4.4), rather than in terms of TCB, the first integral will be computed as

$$\int_{t_0}^{t}\left(\frac{v_E^2}{2} + w_{0ext}(\mathbf{x}_E)\right)dt = \left[\int_{t_{eph_0}}^{t_{eph}}\left(\frac{v_E^2}{2} + w_{0ext}(\mathbf{x}_E)\right)dt_{eph}\right]/(1-L_B) \tag{8}$$

where a rough value of $L_B = 1.5505.. \times 10^{-8}$ may be used.

Terms in the second integral of (7) are secular and quasi periodic. They amount to $\sim 1.10 \times 10^{-16}$ in rate ($\mathrm{d}TCB/\mathrm{d}TCG$) and primarily a yearly term of ~ 30 ps in amplitude (i.e. corresponding to periodic rate variations of amplitude $\sim 6\times10^{-18}$). Besides the position dependent terms in c^{-2} which may reach several microseconds, position dependent terms in c^{-4} (the last two terms in (7)) may reach, for example, an amplitude of 0.4 ps (corresponding to $\sim 3 \times 10^{-17}$ in rate) in geostationary orbit.

To summarize, the IAU'1991 framework is sufficient for applications with an uncertainty not smaller than about 2×10^{-16} in rate or, for quasi-periodic terms, not smaller than a few parts in 10^{17} in rate amplitude. For a smaller uncertainty, Resolution B1.5(2000) provides all the necessary information to perform time transformations in the domain of validity specified in the Resolution (typically 5×10^{-18} in rate). Applications considered are e.g. future space missions carrying high accuracy cold atoms clocks, the frequency comparison of two clocks in two different locations in the solar system or the realization of TCB from an Earth atomic time scale. Finally, when the accuracy specified in Resolution B1.5 will be deemed insufficient, formulas extending those given above should be re-derived from resolutions B1.3 and B1.4.

3.2   Resolution B1.9(2000): Re-definition of Terrestrial Time TT

Evaluating the contributions of the higher order terms in the metric of the geocentric reference system (Resolution B1.3), it is found that the IAU'1991 framework with the metric of the form (1) is sufficient for time and frequency

applications in the GCRS in the light of present and foreseeable future clock accuracies. Although TCG is the time coordinate of the GCRS, presently, the time scale of reference for all practical matters on Earth is Terrestrial Time TT or one of the scales realizing it and differing by some time offset (e.g., TAI, UTC, GPS-time). TT was defined in IAU Resolution A4 (1991) as: "a time scale differing from the Geocentric Coordinate Time TCG by a constant rate, the unit of measurement of TT being chosen so that it agrees with the SI second on the geoid". According to the transformation between proper and coordinate time, this constant rate is given by $\mathrm{d}(TT)/\mathrm{d}(TCG) = 1 - U_g/c^2 = 1 - L_G$, where U_g is the gravity (gravitational + rotational) potential on the geoid.

Some shortcomings appeared in this definition of TT when considering accuracies below 10^{-17}. First, the uncertainty in determination of U_g is of order $1 \text{ m}^2 \text{s}^{-2}$ or slightly better (Groten, 2000). Second, even if it is expected that the uncertainty in U_g improves with time the surface of the geoid is difficult to realize (so that it is difficult to determine the potential difference between the geoid and the location of a clock). Third, the geoid is, in principle, variable with time. Therefore it was decided to desociate the definition of TT from the geoid while maintaining continuity with the previous definition. The constant L_g was turned into a defining constant with its value fixed to $6.969290134 \times 10^{-10}$ to ensure the continuity with the current best estimation of U_G/c^2, from the value $U_G = 62636856 \text{ m}^2 \text{s}^{-2}$ provided by the International Association of Geodesy Special Commission 3 (Groten 2000).

Thus Resolution B1.9 will allow to use the full potential of future high accuracy clocks on board terrestrial satellites to realize TT.

4   Some applications of the IAU'1991+2000 framework

All space geodesy techniques must be considered in the framework of general relativity. The use of a relativistic model implies several steps e.g. as adapted from (Wolf 2001):

1. to choose of an appropriate coordinate system and associated metric, in which to model the technique measurement.
2. to transform all input quantities (measurements + other information e.g. positions, velocities) into this coordinate system.
3. to obtain the results (usually by fitting parameters to measurements) in this coordinate system.
4. to transform the results, if necessary, into proper quantities or into another coordinate system.

The IAU'2000 framework allows to model unambiguously all space geodesy and astrometry techniques using the BCRS and GCRS metric and coordinate systems as defined in Resolution B1.3. It is always possible to define different coordinates e.g. TT is defined as a time coordinate for the GCRS and TDB may be defined as a time coordinate for the BCRS.

$$dTT/dTCG = 1 - L_G, \qquad dTDB/dTCB = 1 - L_B$$

The linear factor between the scale units of the different time coordinates is of order U/c^2 or v^2/c^2 i.e. 7×10^{-10} in the GCRS and 1.5×10^{-8} in the BCRS. It is possible to use the metric of the GCRS or BCRS along with time measurements (input quantities) expressed in another coordinate time but care must be taken when interpreting the results, such as space coordinates or other quantities (e.g. GM). For example, in VLBI analysis the input quantities are generally in TT so that the space coordinates obtained are expressed in "TT

units" in the GCRS (see section 4.1); Or the GM of the solar system bodies are generally obtained from the determination of planetary ephemerides in "TDB units" (see section 4.4). These quantities are related to the true GCRS (TCG) or BCRS (TCB) quantities by the same scaling factors.

For the present time, the approach that seems universally used in space geodesy and astrometry is to transform measurements (input quantities) to TT or TDB, use them in models developed in BCRS or GCRS, and appropriately scale the results, when necessary. One practical reason for this approach is that, in most cases, a quantity of proper time may be directly identified to its value after transformation to a coordinate quantity in TT or TDB. This procedure, when used correctly, may be satisfactory for the present level of uncertainty of the space geodesy or astrometry techniques (e.g. one to a few parts in 10^{10} in relative value). However it offers potential sources of error and confusion and, at some level of uncertainty (possibly of order v^3/c^3 i.e. 10^{-12} in scale or a fraction of μas in angle) it may no longer be adequate. This discussion is purely qualitative and more study will be necessary to precise these possible limitations.

In steps 2 and 4 defined above, one has to perform some of the following tasks: transforming proper quantities to coordinate quantities; transforming between different coordinate quantities; assigning a date (coordinate time) to an event. In section 4.1 to 4.4 below, we review how these tasks are performed for different space geodesy techniques and products and what influence the underlying choice of coordinate system may have on the space geodesy results. Finally in section 4.5 some details are provided on the transformation between TCG and TCB, which is to be used to transform time coordinates when a technique is modeled in the BCRS.

4.1   VLBI delay

The VLBI model (note that we consider here only the vacuum propagation delay and also do not account for the desynchronization or desyntonization of the station clocks) presented in the IERS Standards (1992) and in the draft IERS Conventions (2000) relates the TCG coordinate interval $d_{TCG} = t_2 - t_1$ to a baseline $\vec{b}$ expressed in GCRS coordinates.

$$t_2 - t_1 = \frac{\Delta T_{grav} - \frac{\hat{K}\cdot\vec{b}}{c}\left[1 - \frac{(1+\gamma)U}{c^2} - \frac{|\vec{V}_\oplus|^2}{2c^2} - \frac{\vec{V}_\oplus\cdot\vec{w}_2}{c^2}\right] - \frac{\vec{V}_\oplus\cdot\vec{b}}{c^2}(1 + \hat{K}\cdot\vec{V}_\oplus/2c)}{1 + \frac{\hat{K}\cdot(\vec{V}_\oplus+\vec{w}_2)}{c}} \quad (9)$$

It is not the purpose of this paper to present proper definitions of the quantities used in this formula, which may be found in the mentioned documents. This expression is presented only for evidencing that, with a relative uncertainty of order v/c at least, a scaling in the time coordinate in the left part results in the corresponding scaling of the baseline coordinates in the right part.

In principle, the observable quantities in the VLBI technique are recorded signals measured in the proper time of the station clocks. But, for practical considerations, particularly because the station clocks do not produce ideal proper time (they even are, in general, synchronized and syntonized to UTC to some level, i.e. they have the same rate as the coordinate time TT), the VLBI delay produced by a correlator center may be considered to be, within the uncertainty of the model, equal to the TT coordinate time interval d_{TT} between two events: the arrival of a radio signal from the source at the reference point of the first station, and the arrival of the same signal at the reference point of the second station. In the following, two different approaches are presented to interpret the VLBI delay which use two different geocentric coordinate systems with either TCG or TT as coordinate time.

In the first approach, which could be termed "fully IAU compliant", we consider that all quantities have been transformed to GCRS coordinate quantities i.e. with TCG as a coordinate time. In particular, the VLBI delay obtained from the correlator would have to be scaled to a TCG coordinate interval $d_{TCG} = d_{VLBI}/(1 - L_G)$. The results of the VLBI analysis, i.e. baseline $\vec{b}$ from formula (9) would then be directly obtained in terms of the spatial coordinates of the GCRS, as is recommended by the IUGG Resolution 2 (1991) and IAU Resolution B6 (1997), i.e. one would obtain coordinates that are termed "consistent with TCG", here denoted x_{TCG}.

In the second approach, if the VLBI model (9) is used with VLBI delays as directly provided by correlators (i.e. equivalent to a TT coordinate interval d_{TT} without transformation to TCG), the baseline $\vec{b}$ is not expressed in GCRS but in some other coordinate system. The transformation of these coordinates to GCRS reduces, at the level of uncertainty considered here, to a simple scaling. The space coordinates resulting from the VLBI analysis (here denoted x_{VLBI}) are then termed "consistent with TT" and the coordinates recommended by the IAU and IUGG resolutions, x_{TCG}, may be obtained *a posteriori* by $x_{TCG} = x_{VLBI}/(1 - L_G)$.

4.2   Laser ranging to a satellite or to the Moon

In a reference system centered on an ensemble of masses, as considered in the IAU framework, if a light signal is emitted from x_1 at coordinate time t_1 and is received at x_2 at coordinate time t_2, the coordinate time of propagation is given by

$$t_2 - t_1 = \frac{|\vec{x}_2(t_2) - \vec{x}_1(t_1)|}{c} + \sum_J \frac{2GM_J}{c^3} \ln\left(\frac{r_{J1} + r_{J2} + \rho}{r_{J1} + r_{J2} - \rho}\right) \qquad (10)$$

where the sum is carried out over all bodies J with mass M_J centered at x_J and where $r_{J1} = |\vec{x}_1 - \vec{x}_J|$, $r_{J2} = |\vec{x}_2 - \vec{x}_J|$ and $\rho = |\vec{x}_2 - \vec{x}_1|$.

Because the space coordinates x_i are generally determined from the same data set through the equations of motion of the satellite or of the Moon, it is not so trivial to directly evaluate the impact of the choice of the reference system on the model given by (10). Nevertheless it is possible to specified the coordinate transformations that are to be used in steps 2/4 defined above. The IAU'1991 formalism is sufficient to obtain (10) and to perform the necessary coordinate transformations (it is however not sufficient for the equations of motion, but this subject is not treated here).

For near-Earth satellites (SLR), practical analysis is done in the geocentric frame of reference, and the only body to be considered in the summation of (10) is the Earth (Ries *et al.*, 1988). Measurement of the time of flight of the laser signal is obtained with a clock on Earth, which rate is usually close to that of TT (or corrected to that of TT). The coordinate interval $t_2 - t_1$ is therefore usually taken as a TT interval and the GM value and space coordinates obtained from the dynamical analysis are termed to be "in TT units".

For lunar laser ranging (LLR), which is formulated in the solar system barycentric reference frame, the Sun and the Earth must be taken into account in the summation of (10), with the contribution of the Moon being of order 1 ps (i.e. about 1 mm for a return trip). Moreover, in the analysis of LLR data, the body-centered coordinates of an Earth station and a lunar reflector should be transformed into barycentric coordinates. The transformation of $\vec{r}$, a geocentric position vector expressed in the GCRS, to $\vec{r}_b$, the vector expressed in the BCRS, is provided with an uncertainty lower than 1 mm by the equation

$$\vec{r}_b = \vec{r}\left(1 - \frac{U}{c^2}\right) - \frac{1}{2}\left(\frac{\vec{V} \cdot \vec{r}}{c^2}\right)\vec{V} \tag{11}$$

where U is the gravitational potential at the geocenter (excluding the Earth's mass) and $\vec{V}$ is the barycentric velocity of the Earth. A similar equation applies to the selenocentric reflector coordinates. The time of flight measured by the Earth clock has to be transformed into the corresponding interval of TCB and time tags of measurements expressed in UTC have to be transformed to TCB.

The actual practice of LLR analysis centers is not studied here.

4.3 Scale of ITRF2000

The scale of ITRF2000 is based on "a weighted average of VLBI and most consistent SLR solutions" (draft IERS Conventions 2000). All VLBI analysis centers submitting to the IERS have used the second approach described in section 4.1 and, therefore, the VLBI space coordinates are of the type x_{VLBI}. Similarly all SLR analysis centers perform their analysis using TT as a coordinate time in the geocentric system and presumably provide space coordinates "consistent with TT". For continuity, a ITRF workshop (November 2000) decided to continue to use this approach, making it the present conventional choice for submission to the IERS. At the ITRF workshop, it was also decided that the coordinates should not be re-scaled to x_{TCG} for the computation of ITRF2000 (see IERS Conventions 2000) so that the scale of ITRF2000 does not comply with IAU and IUGG resolutions. Space coordinates "consistent with TCG" may be obtained by $x_{TCG} = x_{ITRF2000} \times (1 + L_G)$.

Note that, contrary to ITRF2000, previous realizations ITRF94 to ITRF97 had been scaled to be in compliance with IAU and IUG resolutions (see IERS Conventions 2000).

Note also that other differences exist between the conventions used to generate ITRF2000 and IAU/IUGG resolutions. They concern mainly the treatment of the permanent tide and the geocenter motion. The IERS Conventions 2000 will provide the formulas to transform coordinates expressed in ITRF2000 to the coordinates that would be obtained in a coordinate system fully compliant with IAU/IUGG resolutions.

4.4 Solar system Ephemerides

The metric tensor of the BCRS, as defined in the IAU'2000 resolutions, allows one to derive (see Soffel et al. 2002) the Einstein-Infeld-Hoffman equations of motion which have been used since the 70s to construct the JPL numerical ephemerides of planetary motion of the DE series (Newhall et al. 1983). These planetary and lunar ephemerides have been recommended for the successive IERS Conventions. For the Conventions 2000, these are DE405 and the Lunar Ephemeris LE405, available on a CD from the publisher, Willmann-Bell. See also the website `http://ssd.jpl.nasa.gov/iau-comm4`; click on the button, "Where to Obtain Ephemerides".

Like for all ephemerides in the DE series, the time scale for DE405/LE405 is not TCB but rather, a coordinate time, T_{eph}, which is related to TCB by an offset and a rate. Because T_{eph} varies with each realization, it can be identified as being essentially the same as, but not exactly equivalent to, TDB (and prior to 1976, ET) (Standish 1998). The GM values of the solar system bodies and the space coordinates obtained from the dynamical analysis are termed to be "in TDB units".

4.5   Transformation between TCG and TCB

The relation between TCG and TCB is given by (2) in the IAU'1991 framework
and by (7) in the IAU'2000 framework. We consider here only the location-
independent part of (2), i.e. the c^{-2} term of the relation between TCG and TCB
at the geocenter, noted $(TCB - TCG)_{c^{-2}}(Geo, TT)$. It is the integral in (2),
i.e. the first integral in (7), and may be computed, through (8), from the Solar
system ephemerides as an integral of the argument T_{eph} of the ephemerides.
This integral in T_{eph} has been termed "time ephemeris" and its current con-
ventional realization is TE405 (Irwin and Fukushima, 1999). It is based on
the JPL ephemerides DE405/LE405 (see previous section) and is available in a
Chebyshev form at `ftp://astroftp.phys.uvic.ca/pub/irwin/tephemeris`.
An analytic formula approximating TE405 to 0.5 ns RMS over 1600-2200 is
in preparation (Fukushima, 2002, personal communication). While the time
argument of TE405 is, strictly speaking, T_{eph}, it may be replaced by TT for
computing TCB-TCG because the error in doing so is at the ps level.

Actually TE405, as it is computed and distributed, here noted $TE405(TT)$,
does not provide directly the integral in T_{eph}, defined above, but this integral
minus a linear term so that

$$(TCB - TCG)_{c^{-2}}(Geo, TT) = (TE405(TT) + \Delta L_C \times (TT - TT_0))/(1 - L_B) \quad (12)$$

where $\Delta L_C = 1.48082685594 \times 10^{-8}$ and where TT_0 corresponds to JD 2443144.5
TAI. The uncertainty is estimated to be of order 1×10^{-17} in rate and a few
ns in periodic terms over 1600-2200.

Analytical formulas which had been developed to compute the transformation
TDB-TDT before 1991 may still be used to compute the TCB-TCG relation.
Among these formulas, one version of the formula developped by (Fairhead and
Bretagnon, 1990) has been made available with the IERS Conventions 1996
(`ftp://maia.usno.navy.mil/conventions/chapter11/fbl.f`). It is refer-
enced in (Irwin and Fukushima, 1999) as FB3B and they have shown that
$FB3B(TT) - FB3B(TT_0)$ is equivalent to $TE405(TT)$ to within $\pm 15ns$ over
1600-2200 and may be used in (12) to this level of uncertainty,

To obtain the full transformation (7) at the geocenter one should also take
into account terms in c^{-4}, as indicated in section 3.2. For completeness, on
may add a small (c^{-2}) contribution from the asteroids which is not included in
$(TCB - TCG)_{c^{-2}}(Geo, TT)$ as computed in (12), an estimate of which is given
in (Irwin and Fukushima, 1999). Finally

$$(TCB - TCG)(Geo, TT) = (TE405(TT) + L_C \times (TT - TT_0))/(1 - L_B) \quad (13)$$

where $L_C = 1.48082686741 \times 10^{-8}$. To obtain the full transformation (7) for
any event, one should also take into account position dependent terms in c^{-2}
(position dependent terms in c^{-4} being generally negligible), as indicated in
section 3.2.

5   Acknowledgement

Discussions with and suggestions from my colleague Peter Wolf are gratefully
acknowledged.

References

Fairhead L., Bretagnon P., *Astron. Astrophys.*, **229**, 240 (1990)

Fukushima T., *Astron. Astrophys.*, **294**, 895 (1995)

Groten E., *Proc IAU Colloquium 180*, 337 (2000)

IAU, Transactions of the International Astronomical Union, **XXIB** (edited by J.Bergeron), Dordrecht, Kluwer (1991)

IERS Standards, International Earth Rotation Service Technical Note 13 (ed. by D.D.McCarthy), Observatoire de Paris (1992)

IERS Conventions, International Earth Rotation Service Technical Note 21 (ed. by D.D.McCarthy), Observatoire de Paris (1996)

Draft IERS Conventions 2000, `http://maia.usno.navy.mil/conv2000.html`

Irwin A., Fukushima T., *Astron. Astrophys.*, **348**, 642 (1999)

Newhall XX, Standish E.M., Williams J.G., *Astron. Astrophys.*, **125**, 150 (1983)

Petit G., *Proc. IAU Colloquium 180*, 275 (2000)

Petit G., *Proc. Journées Systèmes de Référence Spatio-temporels*, 275 (2001)

Ries J.C., Huang C., Watkins M.M., *Phys. Rev. Let.*, **61**, 903 (1988)

Soffel M., *Proc. IAU Colloquium 180*, 283 (2000)

Soffel M. et al., *Astron. J.*, in preparation (2002)

Standish E.M., *Astron. Astrophys.*, **336**, 381 (1998)

Wolf P., *Proc. International School of Physics "Enrico Fermi" Course CXLVI*, 575 (2001)