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VI.2

Gravitomagnetism, Jets in Quasars, and the Stanford Gyroscope Experiment

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William Fairbank is tremendously lucky—or is he clairvoyant? He initiated the Stanford gyroscope experiment in 1961 as a search for the dragging of inertial frames by the earth's rotation—an effect so small as to be interesting in principle, but not in practice. However, today, twenty years later, just when technology development for the gyroscope experiment has reached completion, the theoretical framework for the experiment is changing. Physicists now see the experiment as a search for the gravitational analog of a magnetic field; and astrophysicists now invoke this "gravitomagnetic field" as a power source and alignment force for recently discovered jets squirting out of quasars and galactic nuclei. Suddenly the gyroscope experiment is a crucial test of the mechanism of the most violent explosions in our universe.

In this paper<sup>1</sup> I shall not discuss the gyroscope experiment itself; for that see Everitt [1]. Rather, I shall describe the new astrophysical motivations

<sup>&</sup>lt;sup>1</sup>Some of this paper is adapted, with changes, from §3 of the author's chapter in Quantum Optics, Experimental Gravitation, and Measurement Theory, edited by P. Meystre and M. O. Scully (Plenum Press, New York, 1983).

for it; and while doing so I shall introduce you to some unusual but powerful viewpoints about general relativistic gravity: (i) the split of the spacetime metric  $g_{\alpha\beta}$  and its associated forces into a "gravitoelectric field"  $\mathbf{g}$ , a "gravitomagnetic field"  $\mathbf{H}$ , and a space curvature (not spacetime curvature) with metric  $g_{jk}$ ; and (ii) the "membrane paradigm" for black holes, with its membrane-like event horizon endowed with electric charge, electric current, electric resistance, and an electric battery.

# 1. THE SPLIT OF SPACETIME INTO SPACE PLUS TIME

When spacetime is highly dynamical, for example around two colliding black holes, there is no natural, preferred way to split spacetime up into space plus time. This fact has driven relativists, beginning with Einstein, to describe gravity in terms of a unified, four-dimensional spacetime with dynamically evolving four-dimensional curvature.

On the other hand, astrophysicists and experimental physicists usually deal with situations where spacetime is stationary or nearly stationary rather than dynamical, for example the spacetime around the earth or around a quiescent black hole. In such cases stationarity dictates a preferred way to slice spacetime up into three-dimensional space plus one-dimensional time. Although such "3 + 1 splits" are not treated in standard textbooks on general relativity, they are used widely by professional relativists—for example, in numerical solutions of the Einstein field equations (e.g., Smarr [2]), in the quantization of general relativity (e.g., Wheeler [3]), in astrophysical studies of black holes (e.g., Macdonald and Thorne [4]), and in analyses of laboratory experiments to test general relativity (e.g., Braginsky, Caves and Thorne [5]). There is no approximation inherent in such a 3+1 split; it is merely a rewrite of full general relativity in a new mathematical language.

The 3+1 split regards three-dimensional space as curved rather than Euclidean; its metric  $g_{jk}$  (in an appropriate coordinate system) is just the spatial part of the spacetime metric  $g_{\alpha\beta}$ . In this curved 3-space reside two gravitational potentials: a "gravitoelectric" scalar potential  $\Phi$ , which is essentially the time-time part  $g_{00}$  of the spacetime metric; and a "gravitomagnetic" vector potential  $\gamma$ , which is essentially the time-space part  $g_{0j}$  of the spacetime metric. The decomposition of  $g_{\alpha\beta}$  into  $g_{jk}$ ,  $\Phi$ , and  $\gamma$  is analogous to the decomposition of the electromagnetic four-vector potential  $A_{\alpha}$  into an electrical scalar potential  $\psi = -A_0$  and a magnetic vector potential  $A = A_j$ .

The analogy between gravity and electromagnetism is especially remarkable in the case of systems with weak gravity and low velocities ( $v \ll c$ ),

such as the earth and a gyroscope orbiting it. For such systems a crucial role is played by the "gravitoelectric field"  ${\bf g}$  and the "gravitomagnetic field"  ${\bf H}$ , which are constructed from  $\Phi$  and  $\gamma$  in a manner familiar from electromagnetism:

$$\mathbf{g} = -\nabla \Phi, \quad \mathbf{H} = \nabla \times \gamma, \quad \Phi = -\frac{1}{2}(g_{00} + 1)c^2, \quad \gamma_j = g_{0j} \quad .$$
 (1)

(Here c is the speed of light.) It turns out that g is just the Newtonian gravitational acceleration, and H is a force field of which Newton was unaware because in the dynamics of the solar system its effects are  $\sim 10^{12}$  times smaller than those of g.

For weak gravity, low velocity systems the general relativistic field equations for g and H become almost identical to Maxwell's equations, and the geodesic equation of motion for an uncharged particle is identical to the Lorentz force law (see e.g., Forward [6]; Braginsky, Caves and Thorne [5]):

$$\nabla \cdot \mathbf{g} = \frac{1}{4} 4\pi G \rho, \quad \nabla \times \mathbf{g} = \stackrel{1}{0} ,$$

$$\nabla \cdot \mathbf{H} = 0, \quad \nabla \times \mathbf{H} = \stackrel{1}{4} \left[ -\frac{1}{4} 4\pi G \rho \mathbf{v} / c + (1/c) (\partial \mathbf{g} / \partial t) \right] ;$$
(2)

$$d\mathbf{v}/dt = \mathbf{g} + (\mathbf{v}/c) \times \mathbf{H} \quad . \tag{3}$$

Note that the only differences from Maxwell's equations are (i) minus signs in the source terms (vertical arrows), which cause gravity to be attractive rather than repulsive; (ii) a factor 4 in the strength of  $\mathbf{H}$ , presumably due to gravity being associated with a spin-2 field rather than spin-1; (iii) the replacement of charge density by mass density  $\rho$  times Newton's gravitation constant G; (iv) the replacement of charge current by  $G\rho\mathbf{v}$  where  $\mathbf{v}$  is the velocity of the mass  $\rho$ ; and (v) the absence of  $-(1/c)(\partial \mathbf{H}/\partial t)$  in the  $\nabla \times \mathbf{g}$  equation. In fact,  $-(1/c)(\partial \mathbf{H}/\partial t)$  is absent because I have limited myself to terms which are first order in the v/c of the gravitating mass; including second order terms restores the  $-(1/c)(\partial \mathbf{H}/\partial t)$  but also introduces non-Maxwell-like terms elsewhere in the field equations; see Braginsky, Caves and Thorne [5] for details and for other approximations underlying equations (2) and (3).

## 2. THE EXTERIOR OF A ROTATING SPHERICAL BODY

From our electrodynamical experience we can infer immediately that any rotating spherical body (e.g., the sun or the earth) will be surrounded by

a radial gravitoelectric (Newtonian) field g and a dipolar gravitomagnetic fold. H:

$$\mathbf{g} = -\frac{GM}{r^2}\mathbf{e}_r , \quad \mathbf{H} = \frac{2G}{c} \left[ \frac{\mathbf{S} - 3(\mathbf{S} \cdot \mathbf{e}_r)\mathbf{e}_r}{r^3} \right]$$
(4)

The gravitoelectric monopole moment is the body's mass M; the gravitomagnetic dipole moment is its spin angular momentum S.

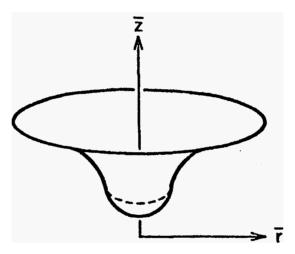


FIGURE 1. Embedding diagram for the equatorial plane of a gravitating spherical body. The curved space outside the body's surface (dashed circle) is described by equations (5) and (6). For a mathematical description of the bowl-like interior see, e.g., [7], pp. 612-615.

The curvature of the space around the spherical body is constant in time and can be described either by the weak-field limit of Schwarzschild's spatial metric

$$ds^2 = (1 + r_g/r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad r_g \equiv 2GM/c^2 \ll r$$
, (5)

or, pictorially, as follows. As we move from one equatorial circle surrounding the body out to another, the circle's circumference increases less rapidly than Euclid would demand:  $d(\text{circumference})/d(\text{radius}) < 2\pi$ . If we imagine extracting the equatorial plane from the curved space of the body and embedding it with unchanged geometry in a flat Euclidean 3-space with coordinates  $\bar{r}, \bar{z}, \bar{\phi}$ , then we obtain the paraboloidal surface (figure 1)

$$\bar{z} = 2\sqrt{r_g(\bar{r} - r_g)}, \quad ds^2 = d\bar{z}^2 + d\bar{r}^2 + \bar{r}^2 d\bar{\phi}^2 ;$$
(6)

cf. pp. 612-615 of Misner, Thorne and Wheeler [7].

Consider a gyroscope with spin angular momentum s in orbit around a rotating earth (the Stanford gyroscope experiment). The three aspects of the earth's gravity [g field, H field, and space curvature; equations (4), (5), (6)] will each produce a precession of the gyroscope relative to the distant stars.

The interaction of the gyroscope's spin s with the earth's gravitomagnetic field H is analogous to the interaction of a magnetic dipole  $\mu$  with a magnetic field B. Just as a torque  $\mu \times B$  acts in the magnetic case, so a torque (1/2)s × H/c acts in the gravitational case. [Equations (1), (2), (4) dictate that  $\mu \to (1/2)s/c$ ,  $B \to H$ .] The gyroscope's angular momentum is changed by this torque:

$$\frac{d\mathbf{s}}{dt} = \frac{1}{2c}\mathbf{s} \times \mathbf{H} \; ; \quad \text{s precesses with the "gravitomagnetic"} \\ \text{angular velocity } \mathbf{\Omega}_{GM} = -\mathbf{H}/2c \; . \tag{7}$$

Note that  $\Omega_{GM}$  is independent of the structure of the gyroscope; this is a manifestation of the principle of equivalence, and it permits one to regard the precession as a "dragging of inertial frames" by the rotation of the earth. This gravitomagnetic precession is often called the "Lense-Thirring" precession, since Lense and Thirring were the first to discover it in the equations of general relativity. For a gyroscope in the 500 km high polar orbit of the Stanford experiment, by averaging  $\Omega_{GM}$  over the orbit and using equation (4) for H, we obtain

$$\Omega_{GM} = \frac{G}{2c^2} \frac{S}{r^3} \simeq 0.05 \frac{\text{arc-seconds}}{\text{year}} ,$$
 (8a)

a precession 50 times greater than the design sensitivity of the Stanford experiment.

The interaction of the gyroscope with the earth's gravitoelectric field g is analogous to the interaction of a classical spinning electron with the Coulomb electric field E of an atomic nucleus. Just as motion of the electron through E induces in the electron's rest frame a magnetic field  $\mathbf{B}_{induced} = -(\mathbf{v}/c) \times \mathbf{E}$  and a torque  $\mu \times \mathbf{B}_{induced}$  and a resulting precession of the electron spin ("atomic spin-orbit coupling"), similarly motion of the gyroscope through g produces in the gyroscope's rest frame a gravitomagnetic field  $\mathbf{H}_{induced} = -(\mathbf{v}/c) \times \mathbf{g}$  and a torque  $((1/2)\mathbf{s}/c) \times \mathbf{H}_{induced}$ and a resulting "spin-orbit" precession of the gyroscope with

$$\Omega_{SO} = -\frac{1}{2c} \mathbf{H}_{induced} = \left[\frac{r_g}{2r}\right]^{5/2} \frac{\mathbf{n}}{r_g/c} \simeq 2.3 \frac{\text{arc-seconds}}{\text{year}}$$
 (8b)

Here  $r_g = 2GM/c^2 = 0.89$  cm is the earth's "gravitational radius," n is the unit normal to the orbital plane, and the orbit is assumed to be 500 kilometers high (r = 6371 + 500 km).

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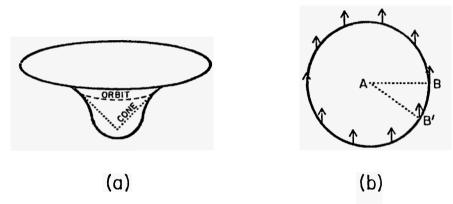


FIGURE 2. Precession of a gyroscope induced by the curvature of space.

In the hydrogen atom there is also a "Thomas precession" which results from the fact that the product of two "velocity boosts" is a combined "boost and rotation." In our gravitational problem the Thomas precession is absent because the gyroscope is presumed to be in a free fall orbit—i.e., it is not accelerated relative to local inertial frames; there are no "boosts." On the other hand, the gravitational field has an aspect, the curvature of space, with no electromagnetic analog; and that space curvature produces an unfamiliar type of precession. In figure 2a we see an embedding diagram (cf. figure 1) for the curved space of the earth. The dashed line is the circular orbit of the gyroscope. The gyroscope can only feel that part of space which is in the immediate neighborhood of its orbit. This allows the pedagogical simplification of replacing the paraboloidal embedding surface of the real curved space by a cone (dotted line) which is tangent to the paraboloid at the gyroscope's orbit. Such a cone can be constructed by drawing a circle on a flat sheet of paper (figure 2b), cutting the pie slice (B-A-B') out of it, and joining the edges A-B and A-B' together. As the gyroscope orbits around the cone it always keeps its spin in the same fixed direction on the flat-sheet-of-paper geometry of the cone's surface (arrows in figure 2b; no local precession). However, as one easily sees by cutting out the pie slice and pasting the cone together, there will be a net precession of the spin when the gyroscope returns to its starting point B = B'. From the shape of the embedding surface [equation (6)] and the cone construction one easily finds that the net precession angle after a single Keplerian orbital period  $(\pi r_g/c)(2r/r_g)^{3/2}$  corresponds to a precession angular velocity

$$\Omega_{SC} = 2 \left[ \frac{r_g}{2r} \right]^{5/2} \frac{\mathbf{n}}{r_g/c} = 2\Omega_{SO} \quad . \tag{8c}$$

Note that this "space-curvature precession" has twice the angular velocity of the spin-orbit precession. The two together bear the name "geodetic precession":

$$\Omega_{geo} = \Omega_{SO} + \Omega_{SC} = 3 \left[ \frac{r_g}{2r} \right]^{5/2} \frac{\mathbf{n}}{r_g/c} \simeq 6.9 \frac{\text{arc-seconds}}{\text{year}}$$
 (8d)

The reader may find it enlightening to compare the above derivations of the relativistic precession formulae with the standard derivation in [7], §40.7.

### 4. THE MEMBRANE PARADIGM FOR BLACK HOLES

As a prelude to my discussion of gravitomagnetic effects in quasars and galactic nuclei, I shall describe the "membrane paradigm" for black holes [18].

"Paradigm" is a word used by the historian of science Thomas Kuhn [8] to describe the body of problem solving techniques, mental pictures and mathematical formalisms used by a specific community of scientists in doing research on a specific set of topics. "Magnetohydrodynamics," with its magnetic field lines like stretchy rubber bands frozen into a plasma, is one paradigm. The "quasilinear theory" of weak plasma turbulence, with its elementary-particle-like plasmons, is another quite different paradigm.

Each paradigm has its own realm of validity and its own regime of computational and conceptual power. Occasionally there occurs a scientific revolution in which an old paradigm (e.g., the "old quantum theory" of pre-19[1424) is replaced by a new paradigm (e.g., the wave mechanics of Schrödinger). On other occasions two very different paradigms exist side by side with a finite domain of overlap—each mathematically equivalent to the other in the domain of overlap, but each having a very different mathematical formalism and set of mental pictures. (Example: magnetohydrodynamics and quasilinear theory with an overlap domain which includes Alfvén waves.)

The theory of black holes was dominated before the mid-1960's by a "frozen-star paradigm," which made extensive use of "Schwarzschild coordinates" with their infinite gravitational redshift at the horizon, and which emphasized mental pictures of collapsing stars that become "frozen" at the horizon because of the redshift. (See, e.g., Zel'dovich and Novikov [9].) A scientific revolution in the 1960's replaced this frozen-star paradigm by the "black hole paradigm," which makes extensive use of "Kruskal coordinates," "Eddington-Finkelstein coordinates," "Penrose diagrams," and

mental pictures of stars that collapse quickly through the horizon and into a singularity. (See, e.g., [7].) Since the mid-1970's a third paradigm has been taking hold. First codified by Damour [10], this "membrane paradigm" (also called "bubble paradigm") treats the horizon of a black hole as a twodimensional membrane in three-dimensional space—a membrane endowed with electric charge, electric current, electric resistivity, an electric battery, viscosity, surface pressure, temperature, entropy and gravitoelectric and gravitomagnetic fields. The membrane paradigm is mathematically equivalent to the black hole paradigm everywhere outside the horizon—i.e., everywhere of relevance for astrophysics. But the membrane paradigm loses its validity inside the horizon. An observer who falls through the horizon discovers, for example, that the horizon is not really endowed with electric charge and current; it merely looked that way from the outside. The membrane paradigm is still under development; for recent work which married it to the 3+1 split of spacetime, see Macdonald and Thorne [4], and Thorne and Macdonald [11]; for a detailed presentation of the paradigm, see [18].

Figure 3 depicts several electromagnetic aspects of the membrane paradigm. At the horizon (membrane) of a black hole the normal component of the electric field  $\mathbf{E}_{\perp}$  is terminated by surface electric charges (charge density  $\sigma_H$ ; Gauss's law), and the tangential component of the magnetic field  $\mathbf{B}_H$  is terminated by surface currents (current density  $\mathbf{J}_H$ ; Ampère's law). Charge is conserved at the horizon: any volume currents  $\mathbf{j}$  flowing into and out of the horizon produce time changes of  $\sigma_H$  and/or divergences of  $\mathbf{J}_H$ . The tangential component of the electric field,  $\mathbf{E}_H$ , is not terminated at the horizon; rather, it extends into the horizon where it drives the surface currents in accordance with Ohm's law:

$$\mathbf{E}_{H} = R_{H} \mathbf{J}_{H} , \quad R_{H} = 4\pi/c = 377 \text{ ohms}$$
 (9)

The normal component of the magnetic field  $\mathbf{B}_{\perp}$  is also not terminated at the horizon; rather, it extends into the horizon where it interacts with the *gravito* magnetic potential  $\gamma$  of equation (1) to produce electric potential drops (battery effect) in the horizon:

$$\int \mathbf{E}_H \cdot d\boldsymbol{\ell} = \Delta V = \int \boldsymbol{\gamma} \times \mathbf{B}_{\perp} \cdot d\boldsymbol{\ell} \quad . \tag{10}$$

This battery will be crucial in the discussion of power sources for quasars, below. (For further details on figure 3 and on equations (9) and (10), including several subtle but crucial issues in the definitions of  $\mathbf{E}_H$  and  $\mathbf{B}_H$ , see Macdonald and Thorne [4] or see [18].)

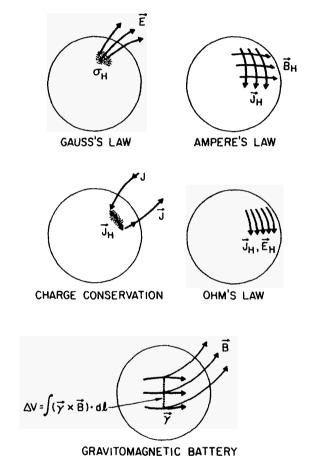


FIGURE 3. Some electromagnetic aspects of the membrane paradigm for black holes.

# 5. GRAVITOMAGNETISM AND RELATIVISTIC PRECESSION

It is now clear that quasars and other strong extragalactic radio sources are fed power ( $P \gtrsim 10^{44}$  erg/sec) by jets of gas and magnetic field, and that each jet is generated by a compact supermassive object ( $M \gtrsim 10^7$  solar masses) in the nucleus of a galaxy; cf. Begelman, Blandford and Rees [12]. A supermassive black hole is the prime candidate for the compact object.

In some radio sources (e.g., NGC 6251, figure 4), the compact object must hold the jet direction constant for times as long as 10 million years. The only way a black hole can do this is by the gyroscopic

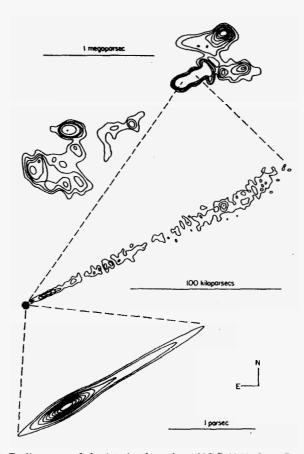


FIGURE 4. Radio maps of the jets in the galaxy NGC 6251; from Readhead, Cohen and Blandford [17].

action of its spin; and the only way it can communicate the direction of its spin to the jet is via its gravitomagnetic field H. In fact, H should produce a gravitomagnetic precession of any accretion disk encircling the hole, and that precession together with the disk's viscosity should drive the inner region of the disk into the hole's equatorial plane (Bardeen and Petterson [13]; figure 5). The resulting configuration has only two preferred directions along which to send jets: the north and south poles of the hole. This mechanism of jet alignment is widely believed by astrophysicists.

Some observed jets precess with precession periods  $\gtrsim 10^4$  years. Astrophysicists attribute this to a precession of the central hole's spin. The most promising way that the spin can be made to precess is by orbital motion of the hole (now acting as a "test gyroscope") around a companion hole

FIGURE 5. The Bardeen-Petterson effect. An accretion disk is driven into the equatorial plane of a black hole.

or other massive object (which acts as a source of g and H fields and of space curvature). The resulting geodetic precession will be faster than the gravitomagnetic precession, and will have a period

$$\frac{2\pi}{\Omega_{geo}} \sim 10^4 \text{ years} \quad \left[ \frac{\text{distance between holes}}{0.01 \text{ parsecs}} \right]^{5/2} \\
\left[ \frac{\text{mass}}{10^8 \text{ solar masses}} \right]^{-3/2} \tag{11}$$

(Begelman, Blandford and Rees [14]).

There are several plausible models for generation of the jets. One of the most attractive (Blandford and Znajek [15]; Macdonald and Thorne [4]) relies on the rotational energy stored in the hole's gravitomagnetic field as the power source, and relies on the horizon's gravitomagnetic battery, equation (10), as that power source's agent.

Consider a rotating black hole surrounded by a magnetized accretion disk (figure 6). As the disk's plasma accretes, it drags magnetic field lines with itself, depositing them on the horizon. Although the **B** fields in the disk may be very chaotic, when deposited on the horizon they quickly slide around ("imperfect magnetohydrodynamics"; "finite conductivity of the horizon"; time scale of sliding  $\sim r_g/c$ ) until they become very orderly. Their ultimate, nearly uniform configuration, as depicted in figure 6, is that which minimizes the horizon's ohmic dissipation (Macdonald and Thorne [4]). These quasiuniform **B**-field lines are held on the hole by Maxwell

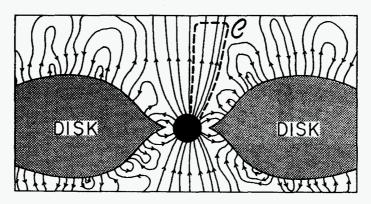


FIGURE 6. Electromagnetic extraction of rotational energy of a black hole ("Blandford-Znajek process").

pressure from the surrounding chaotic field lines, which in turn are anchored in the disk by currents. If the disk were suddenly removed, the field would slide off the horizon, convert itself into radiation, and fly away in a time  $\sim r_q/c$ .

Near the horizon the magnetic field B will be so strong that currents cannot flow across it; but far from the horizon, where B is weaker, they can. Consequently, the B field acts like a dc transmission line. The horizon's gravitomagnetic battery, equation (10), drives currents around closed loops, such as curve C of figure 6: up the **B** field from the horizon to a weak-B region, across the B field there, and then back down the B field to the horizon and through the horizon's battery to the starting point. These currents transmit power in the form of Poynting flux from the horizon to the weak-B region, where it is deposited into charged particles and accelerates them to ultrarelativistic energies. Both the horizon with its gravitomagnetic battery, and the weak-B "acceleration region" with its particles soaking up power, possess total electric resistances of order 30 ohms (Znajek [16]; Damour [10]; Macdonald and Thorne [4]). Thus, the battery and its load are impedance matched, and this "Blandford-Znajek" [15] process has the optimum possible efficiency for depositing the hole's gravitomagnetic, rotational energy into ultra-relativistic charged particles—particles that might well generate the jets observed in quasars and galactic nuclei. Numbers for typical jet models are

$$\begin{bmatrix} \text{Battery} \\ \text{voltage} \end{bmatrix} \sim \frac{1}{4} \left[ \frac{S}{S_{max}} \right] B_{\perp} r_g$$

$$\sim \left[ 10^{20} \text{ volts} \right] \left[ \frac{S}{S_{max}} \right] \left[ \frac{B_{\perp}}{10^4 \text{ gauss}} \right] \left[ \frac{M}{10^9 M_{\odot}} \right] ,$$

$$(12a)$$

where  $S/S_{max}$  is the black hole's angular momentum ("gravitomagnetic dipole moment") in units of the maximum possible angular momentum  $GM^2/c$  of a hole,  $M/10^9M_{\odot}$  is the hole's mass in units of  $10^9$  solar masses.  $B_{\perp}$  is the strength of the ordered magnetic field threading the hole, and  $\mathcal{R}_H = (\text{length/circumference}) \times R_H \sim 30 \text{ ohms is the total electrical resis-}$ tance between the horizon's polar regions and its equatorial regions.

A large number of astrophysicists (not including me) are working on detailed models for the creation and acceleration of jets by the Poynting flux that emerges along a hole's B-field lines, and models for the collimation of the jets by magnetic field tension, by the walls of a funnel-shaped accretion disk, and by nozzles formed in surrounding gas. For a detailed review, see Begelman, Blandford and Rees [12].

### 6. CONCLUSION

It is truly remarkable that the gravitomagnetic potential  $\gamma$  and field H, which are so weak in the solar system that humans have never seen them, are predicted to be so strong near black holes that they are crucial elements in astrophysical model building. As an astrophysicist, I eagerly await the culmination of Bill Fairbank's dreams and planning, and of the decades of brilliant and meticulous technology development by Francis Everitt and others in the Fairbank-Everitt group—a culmination in the human race's first personal encounter with gravitomagnetism and relativistic gyroscope precession.

### Acknowledgments

This work was supported in part by the National Science Foundation (AST82-14126 and PHY77-27084).

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### 586 THORNE

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