Token Economics

Considering "Token Velocity"

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Abstract

Various assertions about the "effect" of token velocity on token exchange rate are considered. The claim that token velocity is an extrinsic quantity that can be throttled to raise token exchange rate, or will naturally increase and lower token exchange rate is examined and rejected. Heuristics are given for future rejection of obviously ridiculous ideas.

Contents

1 Economics in general

"Beware of economists bearing Greek symbols" is generally good life advice[1]. Non-economists attempting to do economics are even more fraught. Anyone attempting to do economics with unclear or incorrect definitions, or with equations that fail basic dimensional analysis or defy laws of addition and division, can be summarily rejected, regardless of who makes the argument.

Economics as a field of thought is occasionally useful, but it is only useful as a *model*: a toy or tool for reasoning about what may or may not happen in an economic system. The power of economic models as tools for *prediction* is fairly low. Even for large, stable and simple economic systems, something like an exchange rate is necessarily going to look like a random walk over most timescales of interest to human beings. For a real world example, consider the dollar exchange rate of Iceland's krona, a "token" for a small economy of about 340,000 people. While there are long term trends, they are not especially predictable, and most of the time the exchange rates look like a random walk. If economic theories were any good at point prediction, there would be no reason for financial services. We could fulfill the dreams of mid-20th century economists such as Bill Phillips[2] and the Soviets[3] to optimally allocate resources without using markets.

Whether to believe the result of a valid economic speculation at all is a matter for human decision makers. Invalid economic ideas can be safely ignored. By invalid economic ideas, I mean ideas that are not internally consistent, use inconsistent definitions, wrong equations, or other such faulty reasoning. The confusion over token velocity and its "influence" on token exchange rate is the result of numerous incorrect economic ideas.

2 Addressing "On Medium of Exchange Tokens"

This "velocipedian" idea seems to originate with a Vitalik Buterin blog post[4] called "On Medium of Exchange Tokens" in October 2017. This blog post contains various overt errors and misunderstandings, which I document below. Quoting relevant sections:

> Traditional macroeconomics has a simple equation to try to value a medium of exchange:
> MV = PT
> Here:
>
> - M is the total money supply; that is, the total number of coins
> - V is the velocity of money; that is, the number of times that an average coin changes hands every day
> - P is the price level. This is the price of goods and services in terms of the token; so it is actually the inverse of the currencys price

> - T is the transaction volume: the economic value of transactions per day
>
> The proof for this is a trivial equality: if there are N coins, and each changes hands M times per day, then this is M * N coins worth of economic value transacted per day. If this represents $ T worth of economic value, then the price of each coin is T / (M * N), so the price level is the inverse of this, M * N / T.

There are several errors here already. Equalities in the world of matter, including the world of economic reality, must have commensurate units. Buterin's assertion is that the total number of units of a token times the velocity of a token (token units per time) is equal to a price in dollars times a price per time (or a price per time). This is false. The definitions are also incorrect. I will correct Buterin's definitions below, starting with Fisher's equation of exchange[5]:

$$P_t^{\mathrm{b}} T_t^{\mathrm{b}} = M^{\mathrm{b}} V_t^{\mathrm{b}}$$

- M^b is the total coin supply of token b
- V_t^b is the velocity of money: i.e., the average number of times that a coin changes hands per unit time, at time t
- P_t^b is the price level. This is the weighted average price of goods and services in terms of the token b at time t
- T_t^b is the quantity of goods/services purchased in terms of tokens b at time t

These definitions are important. The Fisher equation of exchange doesn't make sense without these distinctions. The misunderstanding of the definitions inherent in the equation of exchange causes worse confusion further down:

> For easier analysis, we can recast two variables:
>
> - We refer to 1/V with H, the time that a user holds a coin before using it to make a transaction
> - We refer to 1/P with C, the price of the currency (think C = cost)
>
> Now, we have:
> M/H = T/C
> MC = TH
> The left term is quite simply the market cap. The right term is the economic value transacted per day, multiplied by the amount of time that a user holds a coin before using it to transact.

This is a steady-state model, assuming that the same quantity of users will also be there. In reality, however, the quantity of users may change, and so the price may change. The time that users hold a coin may change, and this may cause the price to change as well.

The inverse of (average) token velocity is not average holding time. For example, let us postulate a money supply of 10 tokens in an economy with a velocity of 10 times per day. If 9 of the tokens are traded once every 1000 days, and one of the tokens 99.991 times a day, this gives mean token velocity 10 times per day. However, the average holding time for a coin in this ecosystem is 900.001 days, not 1/10 day per transaction. Getting things like this right is key to understanding the use of the equation of exchange. The skewed holding time described above is also closer to the present reality for virtually all cryptocurrencies. The distribution of token holding times is skewed, and most tokens are held in expectation of future returns.

Similarly, while Fisher's equation of exchange is an equilibrium model (which I suppose could be called "steady state"), it does not depend on the number of users. There is no number of users in the equation. Where the "quantity of users" comes into play is never revealed; in fact the number of users is irrelevant to the equation of exchange. Number of users can have influence on the variables in the equation, but it doesn't change the result or hinder its applicability. If we assume that average users contribute an approximately fixed amount of economic output to the system, raising number of users will generally grow the economy, but the equation of exchange is valid for an economy of any size.

Finally, the inverse of the weighted average of a token-denoted price is not the exchange rate of a token in terms of dollars. The holding time (as stated above, inverse velocity is not holding time) or the inverse velocity times the token-denominated economic output is also not the market cap in dollars. What is apparently desired here is to come up with something like a conversion rate in dollars per token. It's possible to do this, but the above statements do not do this, and are a mass of confusion and error.

In appendix-3 of the original BAT whitepaper, this operation was done in one line, as I thought writing the actual conversion details were obvious and uninteresting. In hindsight, I realize this was an error in didactics, which may be the origin of the problems above. Starting again from the above Fisher equation of exchange:

$$P_t^b T_t^b = M^b V_t^b$$

If we multiply the left hand side by 1, aka $\frac{P_t^\$}{P_t^\$}$, we get the following relation.

$$\frac{P_t^b}{P_t^\$}(P_t^\$ T_t^b) = M^b V_t^b$$

The term $\frac{P_t^b}{P_t^\$}$ is the number of tokens per dollar (the inverse of the dollar/token exchange rate) $S_b^\$$, therefore:

$$\frac{1}{S_b^{\$}}(P_t^{\$}T_t^b) = M^b V_t^b$$

We can rearrange the values to look like the following, as was done in the whitepaper, though the transformation of the economic output to dollars was swept under the rug:

$$S_t^{\frac{\$}{b}} = \frac{P_t^{\$}T_t^b}{M^b V_t^b}$$

The important thing to note here is that if the token is used to exchange more goods and services *valued in dollars*, the token is also more valuable in terms of dollars.

The denominator is apparently scary to some people, but it can only frighten if you forget what the denominator means. The denominator is just the number of tokens used in exchanges for a given economic system. Let's put it on the other side of the equation where it belongs so we can see what it means:

$$M^b V_t^b S_t^{\frac{\$}{b}} = P_t^{\$}T_t^b$$

The left hand side of the equation is the number of tokens transacted times the conversion rate of tokens to dollars. The right hand side of the equation is the economic output of the token economy converted to dollars. It is a trivial relation. Apparently understanding of this is non-trivial. There is exactly one thing central planners can control here: the number of tokens. The right hand side; economic output, is something that economic participants do. If nobody chooses to use the platform, or the software hasn't been written yet, this value is zero and, as shown below, the exchange rate for a token is an entirely speculative position on future network activity. In a token market which is freely traded, the exchange rate is not set by central planners; it is set by market participants, participants in the token economy, hedgers and speculators.

Belaboring this point further, it helps to go back to Fisher's original statement on the equation of exchange to make it completely concrete and obvious and measured in actual units. We can make a toy example such as in chapter 6 of his book[5], which can be found online. It is an excellent book filled with clear thinking, and should be the first thing read by any would-be economist working with the equation of exchange.

Imagine you are on a desert island with seashells used as money. The island has turtles (cost: four seashells each), coconuts (cost: half a seashell) and seagull eggs (cost: one seashell). In a given month, 200 seagull eggs, 200 turtles and 1000 coconuts are bought and sold. So the total monetary value in seashells for this month is

$$P_t^b T_t^b = 200 \text{ eggs}\frac{1 \text{ seashell}}{\text{egg}} + 200 \text{ turtles}\frac{4 \text{ seashell}}{\text{turtle}} + 1000 \text{ coconut}\frac{0.5 \text{ seashell}}{\text{coconut}} = 1500 \text{ seashell}$$

If our island has a total money supply of 10 seashells, the seashell velocity is 150 in a monthly period. Now, if someone on a motorboat shows up with some large amount of dollars and wants to buy some coconuts (assuming no arbitrage and a \$2 per seashell conversion rate), then

$$P_t^{\$}T_t^b = 1500 \text{ seashell} * \frac{\$2}{\text{seashell}} = M^b V_t^b S_t^{\frac{\$}{b}} = 10 \text{ seashell} * 150 * \frac{\$2}{\text{seashell}}$$

It's obvious from looking at the above that increasing the economic output (i.e., more things bought and sold aka $P_t^{\$}T_t^b$) will make seashells worth more, both in terms of dollars and in local purchasing power. That's the fundamental insight the pricing equation gets us.

Skipping over more false irrelevancies, we come to the source of the velocipedian idea: the 1854 Broad Street cholera outbreak itself.

> Another, and perhaps even more important, conclusion is that the market cap of an appcoin depends crucially on the holding time H. If someone creates a very efficient exchange, which allows users to purchase an appcoin in real time and then immediately use it in the application, then allowing sellers to immediately cash out, then the market cap would drop precipitously. If a currency is stable or prospects are looking optimistic, then this may not matter because users actually see no disadvantage from holding the token instead of holding something else (ie. zero de-facto fee), but if prospects start to turn sour then such a well-functioning exchange can acelerate its demise.

The statement above is false. Again, the inverse of velocity is not average holding time. Worse, the following statement subordinated to his holding time statement is a giant non sequitur. Imagine we go back to my desert island with an economy consisting of seashells, eggs, turtles and coconuts. Now, we're going to crush the seashells into powder and issue 10 SEASHYLL tokens. Because it is a small island, they'll be issued from a central authority. I know the guy who wrote NASDAQ's Xstream crossing engine, so let's say we have a crossing engine as fast as that and it's solar powered, donated by the UN and run feeless by an NGO so it doesn't cost anything to use. Will this increase the velocity of money? Of course it won't. It won't increase if you put it on a perfect and frictionless blockchain either. The only thing that will increase the velocity of a fixed supply of money is if you **increase the economic output of the island.** Maybe you can lure more turtles with the monitor glow from your NASDAQ crossing engine.

It is possible that velocity can be limited technologically, however. If we use, instead of NASDAQ Xstream, a blockchain that crosses one transaction, once a month, everyone on the island will starve to death. This doesn't increase the value of the SEASHYLL token, despite the Fisher equation of exchange; it simply creates an artificial lack of useful money, equilibrium not achieved, and an equation no longer valid. The island dwellers would be better off finding some kind of rock or wampum to use instead of SEASHYLL, as they probably will. Such things have historically happened; the Long Depression of 1873 was caused in part by a lack of useful money, and was in part ameliorated by the issuance of private scrip currencies by banks, the distant ancestors of blockchain money.

There is a price to pay in holding tokens for use in the payment network. Our tokens are potentially long-term deflationary, particularly as stated, if the economy grows. If you use them to buy something, you might be able to buy more things at a later date if you hold the token. As Buterin notices in his essay, this acts like an extra fee on networks. Similarly, short term price volatility might also be considered an extra fee on utility token intermediated networks. Neither is relevant to our use of the equation of exchange. The equation of exchange is not a statement about convertibility of currencies; it is a statement about using a token to mediate economic activity. If we go look at the example of, say, bitcoin, we can immediately see the problem. People are using it as a store of value because it often goes up in value versus the dollar. People also hold volatile tokens in hopes of future return. People also use such tokens to intermediate economic activity where it is useful to do so. Money is useful for buying things, after all. The idea that people will want to cash out immediately in a deflationary or volatile environment just because we have a frictionless exchange somewhere is silly and self-contradictory. People are more inclined to *hold* money when it goes up in value.

3 Addressing assertions in "New Models for Utility Tokens"

Since the above assertions on the subject of velocity are incorrect, there seems little reason to examine subsequent claims by Kyle Samani in his blog post[6] "New Models for Utility Tokens". While the above blog post is a tragedy of misunderstanding, Samani's assertions are a disaster of confusion.

> As I noted in Understanding Token Velocity, the V in the equation of exchange is a huge problem for basically all proprietary payment currencies. Proprietary payment currencies are, generally speaking, susceptible to the velocity problem, which will exert perpetual downwards price pressure. Due to this effect, I expect to see utility tokens that are just proprietary payment currencies exceed a velocity of 100. Velocities of 1,000 are even possible.

There is, as demonstrated above, no "velocity problem." There is also no reason to expect a numeric velocity for an arbitrary utility token made for exchanging goods and services to be 100, 1000 or the square root of the kitchen sink. The two thoughts are not connected. It makes no sense putting the sentences next to each other as if there is some relationship between them.

Rolling back to the cited blog post[7] he mentions, "Understanding Token Velocity", Samani fails to define token velocity correctly in his first mathematical statement:

> Heres where velocity comes in. Its defined as:
> Velocity = Total Transaction Volume / Average Network Value
> Therefore:
> Average Network Value = Total Transaction Volume / Velocity

None of these statements are true. Samani then goes on to make the statement that burning tokens will reduce velocity. This statement is also false. Samani then goes on to make other ill considered assertions about staking, lock up, mint and burn, gamification, etc. Samani also thinks velocity will shrink if you continually shrink the money supply. This is, of course, obviously incorrect.

Going back to the desert island I'd like to leave Samani on, the attentive reader will remember we had a money supply of 10 seashells and a monthly economic output of 1500 seashells, giving a velocity of money of 150 seashells per month. If we were to burn, bury or otherwise somehow lock up 9 of the 10 seashells, the economy is the same size; people still want to trade the same number of coconuts, eggs and turtles. Now, the velocity of money is 1500 seashells per month. When my boat full of dollars shows up, the seashells are of course worth more, which I suppose is what is desired here. But all along for some reason, Samani keeps talking about reducing velocity.

As for the rest of "New Models for Utility Tokens", these are not models. They are varying assertions based on faulty understanding of how economics, ratios or much of anything works. There is no reason to address any of them.

4 Sanity checks

The above exercise is painful and unnecessary.

The lack of self consistent units or definitions in these blog posts means, for any sane person who cares about numbers in the corporeal world, that they may be safely ignored. If your citation is an impenetrable word salad of falsehoods and clodplated babbling, there is no reason to pay attention to the article. It doesn't matter who the author is, as "many people have webpages." As far as can be told, the assertions in these articles were widely, credulously accepted, and nobody, *including the authors,* thought about these ideas in any reasonable or careful way. Performing the seashell exercise would have helped.

The idea that one can raise exchange rates by lowering velocity of money is immediately obviously false. Limiting cases are a useful general technique. Imagine you ran a general payment company that used one token. There is no reason to have more than one token; token size is arbitrary. The company transacts a million dollars in business per day by having a very large token velocity. What happens if you reduce the token velocity to one transaction a year? Is there any way for it to do two million dollars a day in business without increasing token velocity? Will the price of the token go up or down if the payment company does twice the business?

To make the example more obvious: at present, blockchains are constrained to a limited number of transactions per block and a limited number of blocks per year. If we increase the potential number of transactions per block, or provide a micropayment solution (meaning the average token velocity could go up enormously), would we expect the token for the blockchain to go up or down in price? For those who assert that the token for the blockchain would become less valuable in the presence of potential for higher velocity: if this were true, everyone should use blockchains that only cross 10 transactions once per year. Volume limits achieved; we'll all be rich. A blockchain that doesn't function at all, or allow any transactions (aka the velocity will be zero) will be infinitely valuable.

Finally, it is also empirically obvious none of these things apply to utility tokens that exist in the actual world, such as BAT, BTC, 0x, ETH or any other value carrying token. While speculation and correlation with the rest of the cryptocurrency sector has caused a fair amount of volatility to a given token's dollar exchange value, the increased velocity, which has been increasing for the BAT donation system, hasn't caused the price to fall. The same is true for 0x, which has, roughly speaking, appreciated along with the economic transactions in its ecosystem, just as the correct model, as opposed to the ridiculous "inverse velocity model", predicts. The reality of what drives token price is explicated (again) below.

5 A review of our statement on BAT price stability

Speaking of correct economic models, let us review again Appendix-3 of the BAT whitepaper[8].

Our model for virtual currency exchange rates was postulated by Dutch economists von Oordt and Bolt in 2016[9]. The model postulates that the value of virtual currencies consists of three major factors: the utility of the virtual currency to make payments, the decision of forward-looking speculators to regulate the supply of virtual currency, and the elements that drive user adoption and merchant acceptance of a virtual currency.

The argument originates with Fisher's 1911 observation that speculators may effectively limit the money supply by withdrawing money from circulation in anticipation of higher future utility. Since this dynamic particularly applies to limited issuance currencies such as bitcoin or BAT, it can be an important factor in the pricing for token sales and stability analysis of virtual currencies.

For a simple economic system with fixed quantity of currency tokens M^{BAT}, we can write down a transaction quantity relationship:

$$P_t^{\mathrm{BAT}} T_t^{\mathrm{BAT}} = M^{\mathrm{BAT}} V_t^{\mathrm{BAT}}$$

Where V_t^{BAT} is velocity of BAT, the average number of times each unit of BAT is used to purchase services within the defined period of time t. T_t^{BAT} is the quantity of services purchased with BAT over the period of time t and P_t^{BAT} is the weighted price of the services.

Inserting the exchange rate in terms of \$

$$\frac{P_t^{\mathrm{BAT}}}{P_t^{\$}} P_t^{\$} T_t^{\mathrm{BAT}} = M^{\mathrm{BAT}} V_t^{\mathrm{BAT}}$$

Since we can assume the legacy fiat currency is the accounting unit for all parties involved, we define the exchange rate $S_t^{\frac{\$}{\mathrm{BAT}}}$, and substitute in the above equation to give

$$S_t^{\frac{\$}{\mathrm{BAT}}} = \frac{T_t^{\mathrm{BAT}*}}{M^{\mathrm{BAT}} V_t^{\mathrm{BAT}}}$$

Where $T_t^{\mathrm{BAT}*}$ is now considered to be the economic transactions which are done in terms of BAT valued in dollars (aka $P_t^{\$} T_t^{\mathrm{BAT}}$). If we consider the fraction of currency which is not used in transfer of services, we can postulate a velocity of the fraction of currency which is actually used for settlement $\widehat{V_t^{\mathrm{BAT}}}$. Defining Z_t^{BAT} to be the number of BAT units not used in transactions.

Since the entire velocity of money in our economy V_t^{BAT} is an average between the currency units used and the units unused for transfer of services,

$$V_t^{\mathrm{BAT}} = \frac{M^{\mathrm{BAT}} - Z_t^{\mathrm{BAT}}}{M^{\mathrm{BAT}}} \widehat{V_t^{\mathrm{BAT}}}$$

Combining these into the exchange rate

$$S_t^{\frac{\$}{\mathrm{BAT}}} = \frac{\widehat{T_t^{\mathrm{BAT}*}}}{(M^{\mathrm{BAT}} - Z_t^{\mathrm{BAT}}) \widehat{V_t^{\mathrm{BAT}}}} \tag{1}$$

The exchange rate for BAT tokens is therefore proportional to the volume of services purchased and inversely proportional to the currency not used in transactions for the time period t. This equation encapsulates the insight that a lack of money in circulation will raise the exchange rate.

We now turn our attention to the fraction of BAT which is not used for exchange. Some of the Z_t^{BAT} tokens may be the result of users forgetting about the small number of tokens they hold. Some may be due to exchange delays in settlement for legacy currencies. Overall though, the holders of inactive tokens have standard ways of evaluating future utility of the tokens in terms of modern risk management theory.

Rational token holders expect future returns from a position in BAT to be proportional to the volatility of the position over the time period in question, scaled by a risk aversion term γ,

$$\gamma \sigma^2 (S_{t+1}^{\frac{\$}{\mathrm{BAT}}}) z_t^{\mathrm{BAT}}$$

If we consider the future expected exchange rate:

$$||S_{t+1}^{\frac{\$}{\mathrm{BAT}}}||$$

The period t exchange rate is discounted by the risk free exchange rate operator (R) to make it comparable to the future expected exchange rate, we get the time discounted exchange rate:

$$-RS_t^{\frac{\$}{\mathrm{BAT}}}$$

The difference in these two values are equivaluent to the expected returns. We reach the efficient frontier from modern portfolio theory thusly:

$$||S_{t+1}^{\frac{\$}{\mathrm{BAT}}}|| - R(S_t^{\frac{\$}{\mathrm{BAT}}}) = \gamma\sigma^2(S_{t+1}^{\frac{\$}{\mathrm{BAT}}})z_t^{\mathrm{BAT}}$$

Using this standard result, we can solve for the optimal number of tokens held by an individual during a given time period.

$$z_t^{\mathrm{BAT}} = \frac{||S_{t+1}^{\frac{\$}{\mathrm{BAT}}}|| - R(S_t^{\frac{\$}{\mathrm{BAT}}})}{\gamma\sigma^2(S_{t+1}^{\frac{\$}{\mathrm{BAT}}})}$$

If we consider all of the people holding BAT at a given time interval t to have the same risk preferences, we get the economically efficient number of BAT held for later use.

$$Z_t^{\mathrm{BAT}} = N_t z_t^{\mathrm{BAT}} = \frac{||S_{t+1}^{\frac{\$}{\mathrm{BAT}}}||z_t^{\mathrm{BAT}} - R(S_t^{\frac{\$}{\mathrm{BAT}}})}{\frac{\gamma}{N_t}\sigma^2(S_{t+1}^{\frac{\$}{\mathrm{BAT}}})}$$

Since this value can't be negative, we assume that people who hold BAT have the position that expected future exchange rate is

$$||S_{t+1}^{\frac{\$}{\mathrm{BAT}}}|| \geq R(S_t^{\frac{\$}{\mathrm{BAT}}})$$

hence, using our above relationship, we get the relationship between the expected future value of the BAT, the interest rate and the velocity of transfers in the BAT economy:

$$R^{-1}(||S_{t+1}^{\frac{\$}{\mathrm{BAT}}}||) \geq \frac{T_t^{\mathrm{BAT}*}}{M^{\mathrm{BAT}}V_t^{\mathrm{BAT}}}$$

So, people hold BAT if the discounted expected value exceeds the hypothetical value of the current exchange rate. So, the exchange rate as a function of future expected value of BAT is

$$S_t^{\frac{\$}{\mathrm{BAT}}} = R^{-1}(||S_{t+1}^{\frac{\$}{\mathrm{BAT}}}|| - \frac{\gamma}{N_t}Z_t^{\mathrm{BAT}}\sigma^2(S_{t+1}^{\frac{\$}{\mathrm{BAT}}})) \tag{2}$$

Thus, the BAT holdings are the discounted expected future exchange rate minus the risk premium for the uncertainty in future value of the BAT, as expected.

If the model holds, 1 and 2 can be used to define supply and demand for BAT. Since M^{BAT} is not time dependent in the case of BAT, the time varying exchange rate can be readily understood in terms of BAT transactions and opinions on future utility of BAT transactions. As BAT transactions increase, the exchange rate becomes dominated by the transactions rather than future expectations of utility. This dynamic has been observed in maturing virtual currencies as well as various other in-house token systems.

While models are imprecise, this model argues for long term price stability in a token mediated economy.

References

[1] Emmanuel Derman. "Beware of Economists Bearing Greek Symbols". In: *Harvard Business Review* (Oct. 2005). URL: http://www.emanuelderman.com/media/beware.hbr.pdf.

[2] Reserve Bank of New Zealand Museum. *Bill Phillips and the MONIAC*. 2007. URL: https://www.rbnz.govt.nz/research-and-publications/fact-sheets-and-guides/factsheet-bill-phillips-and-the-moniac.

[3] Leonid V. Kantorovich. *The best use of economic resources*. Trans. by P.F. Knightsfield. Pergamon Press, 1965. URL: https://drive.google.com/file/d/1h8VXbMkPRWehB7khZr2lnATdO view.

[4] Vitalik Buterin. *on medium of exchange tokens*. 2017. URL: https://vitalik.ca/general/2017/10/17/moe.html.

[5] Irving Fisher. *The purchasing power of money: Its determination and relation to credit, interest and crises*. Macmillain, 1911. URL: https://www.econlib.org/library/YPDBooks/Fisher/fshPPM.html?chapter_num=6#book-reader.

[6] Kyle Samani. *new models for utility tokens*. 2018. URL: https://multicoin.capital/2018/02/13/new-models-utility-tokens/.

[7] Kyle Samani. *understanding token velocity*. 2017. URL: https://multicoin.capital/2017/12/08/understanding-token-velocity/.

[8] Brave Software International. *Basic Attention Token Whitepaper*. Tech. rep. Mar. 2018. URL: https://basicattentiontoken.org/index/BasicAttentionTokenWhitePaper-4.pdf.

[9] Wilko Bolt and Maarten van Oordt. *On the Value of Virtual Currencies*. Tech. rep. Working Paper No. 2016-42. Bank of Canada, Apr. 2016.

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