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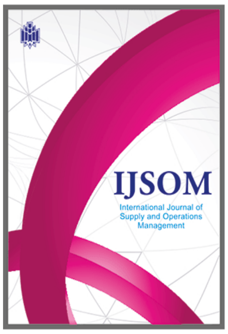


# Modeling and Solving the Multi-depot Vehicle Routing Problem with Time Window by Considering the Flexible End Depot in Each Route

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**Abstract**
This paper considers the multi-depot vehicle routing problem with time window in which each vehicle starts from a depot and there is no need to return to its primary depot after serving customers. The mathematical model which is developed by a new approach aims to minimize the transportation costs including the travelled distance, the latest and the earliest arrival time penalties. Furthermore, in order to reduce the problem searching space, a novel GA clustering method is developed. Finally, Experiments are run on a number of problems with varying depots and time window, and customer sizes. The method is compared to two other clustering techniques, fuzzy C means (FCM) and K-means algorithms. Experimental results show the robustness and effectiveness of the proposed algorithm.

**Keywords:** Vehicle Routing Problem; Multi-depot; Flexible End Depot; Genetic Algorithm; Clustering.

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## 1. Introduction

In the recent decades, the vehicle routing problem (VRP) seems to be one of the main subjects which have been attracted too much attention to increase the efficiency and profitability of transportation systems. VRP is defined as the problems in which the vehicle fleets deployed in one or more than one depot. These fleets are applied to serve a number of customers in predetermined geographical locations. The objective of VRP is to determine the appropriate routes to serve customers, with the minimal cost [Toth and Vigo, 2002].

Vehicle routing problem (VRP) is categorized to different types of problems such as: vehicle routing problem with time windows, periodic vehicle routing problem, and vehicle routing problem by pick up/delivery. Also the VRP can be Single Depot (SD) or Multi Depot (MD). The focus of this research would be on Multi-depot vehicle routing problem with time windows (MDVRPTW).

(MDVRPTW) is a special kind of vehicle routing problem that is highly used in practice. Since it emphasizes on the time subject in solving problem and also considers multi-depot instead of a single depot, it can be more realistic and efficient. In this problem vehicle starts and ends at depot by serving a number of customers (in a defined time windows for each one).

In the MDVRP, the customers are served by several vehicles; each one is located in one of the several depots established in different places. Besides, the MDVRP is NP-hard which means that an efficient algorithm to solve the problem to optimality is unavailable. Therefore, solving the problem by an exact algorithm is time consuming and computationally intractable so in order to deal with this category of problems heuristic and meta-heuristic algorithms are used [Mirabi., Ghomi.. F, and Jolai.2010].

In the VRPTW, Each customer i should be served in his time windows that is defined as $[e_i, l_i]$ in which $e_i$ is the earliest arrival time and $l_i$ is the latest arrival time to serve customers. If the customer does not receive service in time windows, the vehicle will be fined [Tan, Lee, Zhu and Qu. 2001].

In this research, the MDVRPTW is analyzed in which the first and the last depots of each route are different and there is no need to return the vehicle to the primary depot after going out of one depot and serving to customers. But, it goes to the depot, nearest to the last visited customer. The main idea of this method was proposed by Kek et al [2008] and also, Eidi & AbdulRahimi [2012] applied this idea to solve vehicle routing problem in multi-depot and multi-period. The objective of these researchers is to optimize a set of routes for vehicle in order to serve all customers in their defined time windows and as a result, the travelled distance and the transportation cost are reduced by considering the earliest and the latest arrival times without violating the capacity of vehicles. The rest of this paper is organized as follows. In Section 2 we review the literature and Section 3 represents the model formulation and the problem-solving methodology and the computational results are discussed in Section 4. Finally, the conclusion is presented in Section 5.

## 2. Literature review

In this section major studies MDVRP, VRPTW and also MDVRPTW are introduced and analyzed.

One of the first studies about MDVRP pertains to Renaud et al (1996). They proposed the tabu search to deal with the MDVRP. After that Cordeau et al (1997) adopted the tabu search heuristic algorithm to solve periodic MDVRP. Ho et al (2008), proposed two genetic algorithms to deal with the MDVRP that each one had different solutions to the problem. Finally, they compared the performance of two algorithms. Mirabi et al (2010) introduced three heuristic techniques to optimality for MDVRP. Salhi and Sari (1997) proposed a heuristic method with three levels to solve MDVRP. The first level was the construction of a feasible solution and the second and the third levels were to improve the routes in each depot and finally Crevier et al (2007) considered the MDVRP in a way that vehicles can complete their capacities in the middle depots.

The other concept focused in this article is time windows. This problem is one of the main hybrid optimization problems that recent researchers have considered in distributing goods and services. The VRPTW consists of a set of vehicle routes in which vehicles start from a central depot to serve a number of customers with different demands in a defined time windows and finish at the first depot. Amongst the main studies in this field some are as follows: Thangiah in (1994) and Tan et al in 2001 analyzed various heuristic methods on VRPTW and compared them. Ombuki et al (2006) adopted a genetic algorithm and pareto classes to solve the multi-object VRPTW. In this case, the objective is to minimize the number of vehicles and the total costs (distances). Alvarenga et al (2007) applied a genetic algorithm and dealt with this problem by dividing two-phase sets and compared the results to the exact and heuristic methods which were published in advance and indicated that the offered heuristic method was better than the previous ones. Kallehauge (2008) analyzed the exact algorithms which were used to solve vehicle routing problem with time windows in recent decades. Yu & Yang (2011) began to solve periodic vehicle routing problem with time windows by meta-heuristic ant colony algorithm. Banos et al (2013) began to solve VRPSTW problem by simulated annealing algorithm based multi-object parallel approach (MT-PSA). Finally, Kritikos & Ioannou (2010) analyzed the vehicle routing problem with time windows which aimed at balancing load delivered by every active vehicle in fleet and then they applied data covering analytic method to deal with their problem.

There have been few projects on the MDVRPTW problem since its first proposal. These projects are as follows:

Dondo and Cerda (2007) applied the clustering method to deal with VRPTW, firstly by the single depot and homogenous vehicle and secondly by multi-depot and heterogonous vehicle. Also in other research, Dondo and Cerda (2009) considered a local search algorithm. They began to analyze a big neighborhood to get the feasible routes. This neighborhood structure consisted of all solutions which could be offered by reducing the problem and a plan of locational analysis was used. Finally, Xu et al (2012) applied variable neighborhood search to deal with the MDVRPTW problem. In this algorithm, a hybrid operator of insertion and exchange was used to get to a shaking process. Therefore, the strategy of the best improvement was established and it could make it possible for the algorithm to balance the quality of solutions with the time of execution better than before.

## 3. Proposed Method

In this article, we try to serve some customers with determined demands from several depots by a number of vehicles. But, here, we have some constraints: the constraint of the vehicle capacity and time constraint.

On the other hand, the neighborhood or adjacent customers should be served by one vehicle to reduce costs. So, we try to divide the customer in groups. According to the clustering algorithms mentioned in earlier section, the customers are clustered based on time and capacity constraints. Then, we try to design the shortest and the best route for the customers. After problem formulation, we use a genetic algorithm to find the best route to visit customers. The mathematical modeling and proposed genetic algorithm are described as follows:

### *3.1. MDVRP formula, equation and constraints*

Although, the vehicle routing problems are NP-hard, they can be formulated as an integer mathematical scheduling model. Here, we propose: hypotheses, indexes, decision making variables and the mathematical model of the multi-depot vehicle routing problem by considering time windows. The most important hypotheses of this problem are:

- The number of available vehicles is predetermined.
- The capacity of vehicle is defined and fixed.
- The number and location of depots is defined earlier.
- Each vehicle starts from a depot and there is no need to return to the same depot in other words the finished depot can be different from the started depot.
- The number and location of customer are predefined.
- The speed of the vehicle is fixed.
- The transportation cost of each vehicle depends on the travelled distance.
- The transportation network is considered symmetrical.

**Index**

$i$: Customer index

$j$: Customer index

$k$: Vehicle index

$d$: Depot index

**Parameters**

$C_{ij}$: The transportation cost from customer *i* to customer *j*

$C'_{di}$: The transportation cost from depot *d* to customer *i*

$d_i$: Demand of customer *j*

N: the set of customers
B: subset of customers

$Q_k$: Capacity of vehicle *k*

V: the set of Vehicle
D: the set of depots
G: set of all customers and depots
M: positive large number

$e_i$: Earliest service time at node *i*

$l_i$: Latest service time at node *i*

$T_i$: Vehicle arrival time at node *i*

$\rho_i$: Penalty cost for unit-time violations of the specified time window for node *i*

**Variables**

$\Delta a_i$: i$^{th}$-time window violation due to early service

$\Delta b_i$: i$^{th}$-time window violation due to late service

$X_{ijk}$: 1, if vehicle k travels directly from customer *i* to customer *j* (*i*, *j* ∈N); 0 otherwise

$Y_{dik}$: 1, if vehicle k travels directly from depot *d* to customer *j* (i, j ∈N); 0 otherwise

$Z_{idk}$: 1, if vehicle k travels directly from customer *i* to depot *d* (i, j ∈N); 0 otherwise

$$Min \quad \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\sum_{k=1}^{V} X_{ijk}C_{ij} + \sum_{d=1}^{D}\sum_{i=1}^{N}\sum_{k=1}^{V} Y_{dik}C'_{di} + \sum_{i=1}^{N}\sum_{d=1}^{D}\sum_{k=1}^{V} Z_{idk}C'_{di} + \sum_{i=1}^{N}\rho_i\left(\Delta a_i + \Delta b_i\right) \tag{1}$$

$$\sum_{d=1}^{D}\sum_{k=1}^{V} Y_{dik} + \sum_{\substack{j=1\\ j\neq i}}^{N}\sum_{k=1}^{V} X_{jik} = 1 \qquad \forall i \tag{2}$$

$$\sum_{\substack{j=1\\ j\neq i}}^{N}\sum_{k=1}^{V} X_{ijk} + \sum_{d=1}^{D}\sum_{k=1}^{V} Z_{idk} = 1 \qquad \forall i \tag{3}$$

$$\sum_{d=1}^{D}\sum_{i=1}^{N} Y_{dik} - \sum_{j=1}^{N}\sum_{d=1}^{D} Z_{jdk} = 0 \qquad \forall k \tag{4}$$

$$\sum_{d=1}^{D}\sum_{i=1}^{N} Y_{dik} d_i + \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} X_{ijk} d_j \leq C_k \qquad \forall \mathrm{k} \tag{5}$$

$$\sum_{d=1}^{D} Y_{dik} + \sum_{\substack{j=1\\ j\neq i}}^{N} X_{jik} - \sum_{\substack{j=1\\ j\neq i}}^{N} X_{ijk} - \sum_{d=1}^{D} Z_{idk} = 0 \qquad \forall k, i \tag{6}$$

$$\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} X_{ijk} \leq \left(\sum_{d=1}^{D}\sum_{i=1}^{N} Y_{dik}\right) * M \qquad \forall k \tag{7}$$

$$\sum_{i\in B}\sum_{\substack{j\in B\\ j\neq i}} X_{ijk} \leq |B| - 1 \qquad \forall k \qquad \forall B \subseteq G \setminus \{A\} \ , \ |B| \geq 2 \tag{8}$$

$$\Delta a_i \geq e_i - T_i \qquad \forall i \tag{9}$$

$$\Delta b_i \geq T_i - l_i \qquad \forall i \tag{10}$$

$$X_{ijk} \in \{0,1\} \qquad \forall i, k \quad i \neq j \tag{11}$$

$$Y_{dik} \in \{0,1\} \qquad \forall i, k, d \tag{12}$$

$$Z_{idk} \in \{0,1\} \qquad \forall i, k, d \tag{13}$$

The objective function consists of four parts which are as follow: the necessary cost to travel among customers, the necessary cost to travel between the depot and the first customers and the necessary cost to travel between the last customers and depots in each route and finally the last one, the cost of not serving the customers on time which should be minimized. The constraints (2) and (3) ensure that each customer should be visited once in every period of time. The constraint (2) reveals that an arc cannot be travelled by a vehicle, unless this vehicle starts from the single depot and it causes a customer to be at the beginning of the route after a depot or after the other customer. In constraint (3), this route consists of the route of customer to the other customer and customer to the depot i.e. each customer can either link to a depot or a customer. These two constraints lead to serve all customers. The constraint (4) is related to the start and finish of each route and certify that each route starts from the single depot and finishes at the single depot.

The constraint (5) is related to the capacity of each vehicle in such a way that the total demands of the customers don't violate the capacity of the vehicle. The constraint (6) implies that the input and output of the customers are equal and each customer receives services once. The constraint (7) ensures that an arc cannot be travelled by the vehicle unless this vehicle starts from the single depot. The constraint (8) restrains generating sub tour. The constraint (9) states all available vehicles should be used.

The constraints (10) & (11) are related to time windows. The constraints of (12) to (14) are related to the authorized amount for model decision making variables which are all limited to zero and one.

### *3.2.MDVRPTW routing by GA*

The genetic algorithm is a technique that uses genetic evaluation and Darwin’s natural selection principles as a problem solution. The natural selection rule expresses that the generations who have the best characteristics and these superior characteristics are transferred to the subsequent generations. these generations can survive and the other species would extinct gradually. The primary concepts of genetic algorithm were first proposed by Holland (1975) theoretically. This algorithm works with a set of chromosomes named the primary population that each one indicates an answer to the supposed problem. They bear the crossover and mutation operators and produce offspring by a special approach of selecting parents and replacing children. This algorithm is reiterated enough so that the number of offspring production (reiterations) approaches to the defined amount or there is no improvement in the new produced population.

### *3.2.1. Initial population*

The possible depot and routes are produced randomly for each chromosome. The numbers of allocated customers to the first vehicle depends on the vehicle capacity and time Constraint. The arrangement of the customers in chromosomes is in the way that it does not violate the time windows constraint of each customer; otherwise, they will be fined. We aim at minimizing these penalties to reduce distribution costs. In this step, we produce chromosomes based on the predetermined population size number in the algorithm. Each answer of this algorithm is one chromosome.

For example, we show routing 2 vehicles, 10 customers and 3 depots in below figure.

| Primary depot for vehicle1 | Arrange to visit customers | | | Ending depot of route for vehicle1 | | Primary depot for vehicle2 | Arrange to visit customers | | | | | Ending depot of route for vehicle2 | | Primary depot for vehicle3 | Arrangement to visit customers | | End ing depot of route for vehicle3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 4 | 5 | 3 | * | 2 | 2 | 3 | 7 | 8 | 10 | 2 | * | 2 | 6 | 9 | 3 |

### *3.2.2. Selection*

Selection is a method in which chromosomes are extracted randomly from population according to their fitness computation, so that hybridization is done. There are many different selection methods such as roulette wheel and rank selection for a few names [Sivanandam and Deepa.2008]. In Rank Selection, the population is classified and each chromosome is given a fitness value based on its class. The worst fitness value is 1 and the best one catches N value. This leads to decelerating convergence process but it retains premature convergence, too. It also causes the selection pressure remain up when the variance is low and it holds variety and follows a successful search. In fact, the parents with the potential talent are selected and they compete to decide which one is qualified to be a parent. Therefore, rank selection is used in this study.

### *3.2.3. Crossover*

In the offered genetic algorithm, the PMX method is used for crossover [Sivanandam and Deepa.2008]. In this method, two points are randomly selected as crossover points. Then, the space among these cross points is swap in two chromosomes and, two sides are given values. To do this, following steps are proposed:

Step (1): the main parameter of the hybrid operator, i.e., hybrid value is regulated.

Step (2): two crossover points are selected randomly in the part related to visit customers.

P1= (1 2 3 |4 5 6 7 |8 9 10)
P2= (4 5 2 |1 8 7 6 |9 3 10)

Step (3): to produce offspring, the genes among two crossover points are replaced.

O1= (- - - |1 8 7 6 |- - -)
O2= (- - - |4 5 6 7|- - -)

Step (4): in the first parent, we start to analyze from the beginning of the chromosome and each number which is not among the first child's chromosome should be written exactly in its related gene and there should be some blank spaces for the reiterated numbers and then, we do the same processes for the second child.

O1= (- 2 3|1 8 7 6 |- 9 10)
O2= (- - 2 |4 5 6 7|9 3 10)

Step (5): then, to fill the blank spaces in the first child, we start from the beginning of the chromosome in the second parent and each number which is not in the first child is put in the blank space, we continue this process to fill all blank spaces. For the second child, we start from the beginning of the chromosome in the first parent and each number which is not in the second child is put in the blank space.

O1= (4 2 3|1 8 7 6 |5 9 10)
O2= (1 8 2 |4 5 6 7|9 3 10)

### *3.2.4. Mutation*

In this section, the insertion method (Insert mutation) has been used for the acts of the mutation operator as follows:

Step (1): Regulate the mutation value which is proportional to the problem.

Step (2): A single route is selected randomly.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Step (3): Two numbers related to the customers' gene are considered randomly from the selected route in the previous step.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Step (4): the second gene or the second number is placed after the first gene, exactly. The entire next genes other than these two are taken to the next place.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 6 | 3 | 4 | 5 | 7 | 8 | 9 | 10 |

### *3.2.5. Stopping Criterion*

The algorithm will finish, if a defined number of generations is obtained. The new generation recreation will continue until the number of generations does not violate the maximal value. The results obtained from executing the modal on some exemplified problems are proposed later and in the next section.

## *3.3.Clustering*

First, in this article, data (customers) are divided into groups by the clustering method and then we offer a genetic algorithm which is used to find the shortest route amongst customers. In this study, GA-clustering method is used to cluster customers and is compared to FCM and K-means clustering methods. Then, the offered genetic algorithm for finding the shortest route amongst customers is applied for each one. First, we should explain clustering procedure.

Clustering is a technique of dividing to the similar groups or things and each group is called a cluster. Things in each cluster are similar to one another, but they have nothing in common with the existing things in other clusters. Usually, data are caught as vectors in a multi-dimensional space and then they are put in groups or clusters. Therefore, we should find similar groups of things to put them in a cluster. These algorithms have various forms, but the process is reiterated in all those which want to estimate the below.

to find the centers of the clusters, the Euclidean distance of Z&X is calculated as follows and Z is the center of the cluster:

$$D = \| x - z \|$$

Then, each instance data is proportioned to a cluster, while, data has the shortest distance to the center of the cluster. The shorter the distance, the more similarity is achieved in the clustering in the N-dimensional Euclidean space of the same partitioning $N^{th}$ point to *K* under the space (cluster) and is based on similarities and differences. This n^th^ point is shown as the set of S that includes K clusters of $c_1, c_2, c_3, \dots, c_k$. So, we have:

$$c_i \neq 0 \, for \, i = 1,2,\dots,k$$

$$c_i \cap c_j = 0 \, for \, i = 1,2,\dots,k \quad ,j = 1,2,\dots k \, and \, i \neq j$$

$$And \bigcup_{i=1}^{k} c_i = s$$

In the next section we explain about the clustering method used in this research.

*3.4.GA-clustering Algorithm*

In GA clustering a benchmark clustering which is called μ is defined as follows:

$$\mu(C_1, C_2, \dots, C_k) = \sum \sum \|X_i - Z_j\|$$

GA should find the fit cluster centers $Z_1, Z_2, Z_3, \dots, Z_k$ in a way to minimize the benchmark clustering μ (minimize) [10]. The basic steps of GA algorithm, GA clustering algorithm can be represented as follows.

*3.4.1. Population Initialization*

One of the main steps in this algorithm is formation of chromosome. In this step, K center of the cluster is selected randomly and consist of initial population. Therefore, a set of initial answers is produced as a set of chromosomes. The length of each chromosome is N*K in a N-dimensional space.

*3.4.2. Fitness Computation*

Fitness Computation consists of two steps. First, the clusters are formed by the centers encoded in chromosomes. In this way, each $X_i$ point is allocated to one of the clusters $c_i$ with center $Z_i$ given that:

$$\|X_i - Z_j\| < \|X_i - Z_p\| \, ,p = 1,2,\dots,k \quad ,and \, j \neq p.$$

After clustering, the centers of the clusters presented in chromosomes are replaced by the mean of each cluster's points. The new center $Z_i$ for $c_i$ cluster is obtained by the following equation that replaces the previous center:

$$Z_i^* = {}^1/_{n_i} \sum X_j \quad , i = 1,2, \dots, k$$

*3.4.3. Selection*

We have applied the Rank selection method to choose parents as selection method in the genetic clustering.

*3.4.4. Crossover*

This operator is the main operator in a genetic algorithm which has the main role in convergence and it approaches the optimization. Hybridization is a probable process in which a couple of chromosomes considered in the selection step combine to produce the new generation of chromosomes. There are many hybridization methods for problems encoded as a string of numbers such as one-point, two-points, heuristic methods and merge. Here, the crossover method is used by the fixed hybrid value$P_c$.

*3.4.5. Mutation*

This genetic operator works on one chromosome, refining a part of one chromosome, a new chromosome is produced and each chromosome participates in mutation with a fixed probability. Here, as crossover operator, the randomization of the operator plays a special role. The objective of executing this operator is to preserve the variety in population, to find more space and to restrain the premature convergence. The crossover operator can hybridize parent's characteristics and the mutation operator should produce new characteristics. Therefore, each of two operators is vital in the algorithm. The Mutation method used in this paper is described by [Sivanandam and Deepa.2008].

*3.4.6. Stop condition*

In this step, when the fitness, crossover and mutation computation processes are reiterated to the maximum, the algorithm will stop.

## 4. Computational results

Furthermore, in the paper, proposed GA-clustering method is compared to two other algorithms, FCM and K-means. Therefore, a brief description about them is presented as follows.

**K-means**

This method is based on clustering proposed by MC Queen (1967). In general, the K-Means method can be summarized in some steps:

Step (1): first, we choose a point randomly between n-point K data as $Z_1, Z_2, Z_3, \dots, Z_k$ as the centers of the clusters.

Step (2): i=1, 2, 3,… , n and $X_i$ are allocated to the supposed cluster $c_i$ if and only if:

$$\|X_i - Z_j\| < \|X_i - Z_p\| \ , \quad p = 1,2,\dots,k \quad ,andj \neq p.$$

Step (3): first, all data are allocated to the clusters, and then the new centers of $Z_1, Z_2, Z_3, \dots, Z_k$ cluster are calculated as follow:

$$Z_i^* = {}^1/_{n_i} \sum X_j \quad ,i = 1,2,\dots,k \qquad X_j \in C_i$$

$n_i$ is the number of elements allocated to the $c_i$ cluster.

Step (4): step (2) & (3) are reiterated until there are no changes in the centers of the clusters, it means that if$i = 1,2,\dots,k.\quad Z_i = Z_i$Algorithm finishes, if not, it continues from step (2).

When the process does not finish in the step (4) naturally, it is better to execute the above process per maximum number of predetermined iterations [Hosseininezhad and Salajegheh .2012].

**Fuzzy c- means (FCM)**

The definite algorithms of the clusters are partitioned in a way that each data is exactly allocated to the single cluster. In some cases, each data cannot be exactly allocated to a single cluster, because, some data are amongst the clusters. In these cases, the fuzzy clustering methods are better means to show the real structure of this data. In this technique, each thing is allocated to the clusters by a degree of join, so, a single thing can be the member of two or several clusters simultaneously.

The degree of membership is between 0 and 1, and sum of the degree of membership of a single thing in various clusters equal 1. Parameter $\rho$ controls the fuzzy amount in clustering. The less this parameter is, the lesser the fuzzy amount would be and the closer to the infinity is, the more the fuzzy amount would be [Hosseininezhad and Salajegheh .2012].

In this section, we compare the results obtained from the clustering algorithm GA, K-means and FCM algorithms by using some solved examples of Cordeau problem. In all subsequent methods (after clustering), routing is done by a genetic algorithm. The total travelled distance by vehicle is considered per kilometer to serve all customers in order to compare the answers obtained from the algorithms in this research exactly. The number of customers, depots and vehicles have been defined in each kind of problem. In order to show the effectiveness and robustness of the proposed algorithm, simulation has been done on salmon dataset including 16 categories of data for MDVRPTW problem.

**Table 1.** A comparison of total distances with GA-clustering and k-mean and FCM algorithms

| The kind of problem | The number of customers | The number of vehicles | The number of depots | The answer obtained from the GA | The answers obtained from the k-Means | **The answers obtained from the FCM** |
|---|---|---|---|---|---|---|
| P01 | 48 | 2 | 4 | 1052 | 1106 | **1249** |
| P02 | 96 | 3 | 4 | 2401.7 | 2716.7 | **2958.5** |
| P03 | 144 | 4 | 4 | 2352.4 | 2615.7 | **2616.7** |
| P04 | 192 | 5 | 4 | 2501.4 | 2752.8 | **2716.1** |
| P05 | 240 | 4 | 6 | 2292.8 | 2832.2 | **2837.6** |
| P06 | 288 | 2 | 6 | 3011.3 | 3055.7 | **3495.8** |
| P08 | 144 | 3 | 6 | 3146.8 | 3238.5 | **3334.2** |
| P09 | 216 | 4 | 6 | 2988.6 | 3207.5 | **3203.3** |
| P10 | 288 | 5 | 6 | 2695.6 | 2703.5 | **2804** |
| P12 | 96 | 2 | 4 | 2924.6 | 3512.2 | **2986** |
| P13 | 144 | 3 | 4 | 3682.9 | 3697 | **3929** |
| P14 | 192 | 4 | 4 | 2802 | 2885.2 | **2889** |
| P15 | 240 | 5 | 4 | 2494.5 | 2647.1 | **2673.5** |
| P16 | 288 | 6 | 4 | 2392.4 | 2498.6 | **2516.9** |
| P18 | 144 | 2 | 6 | 3623.8 | 3703.8 | |
| P19 | 216 | 3 | 6 | 3350.2 | 3451.7 | **3429.7** |
| P20 | 288 | 4 | 6 | 2870.1 | 2933.1 | **3001.5** |

The obtained answer in all instances mentioned above, are the best obtained ones in 1000 generation recreation of the offered algorithm. According to the table (1), the answers obtained from the algorithms indicate that the GA-clustering method is better than clustering other methods. In figures (1), (2) and (3) the output instance has been show for p03 problem in each algorithm.

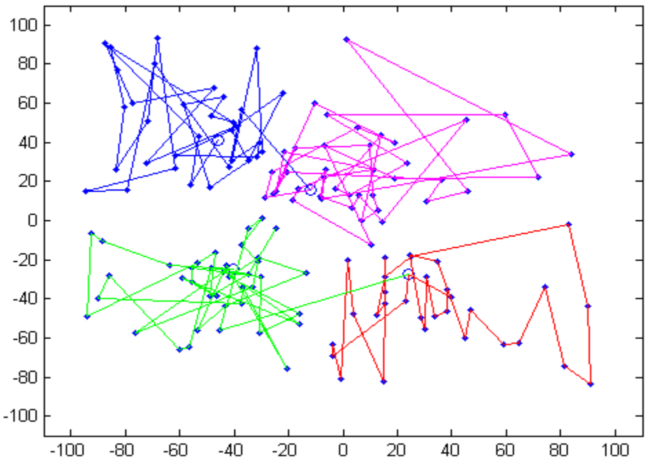


**Figure 1.** The optimum route in the K-means method in p03

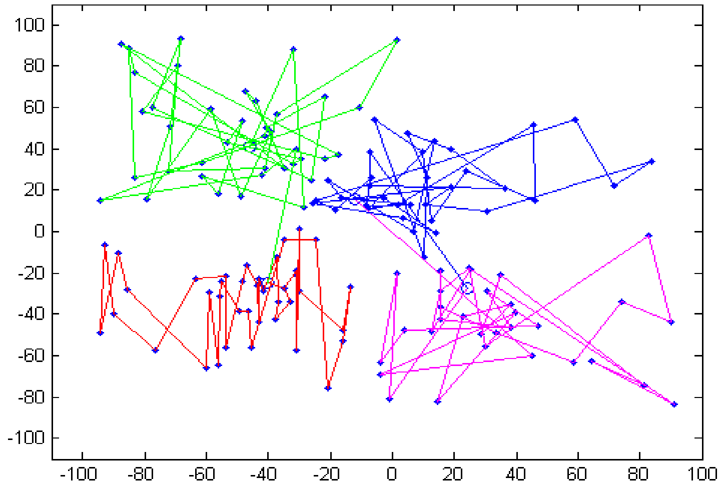


**Figure 2.** The optimum route in the GA method p03

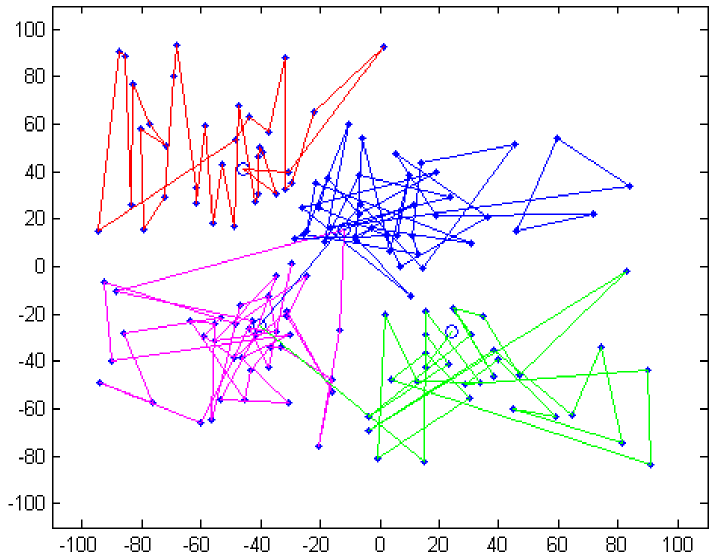


**Figure 3.** The optimum route in the FCM method p03

As one can see, in the obtained figures, there are clusters equal to the number of vehicles.

As stated in formulation of the problem, in this research we consider the case that each vehicle in the end of the route doesn't need to return to the primary depot, and arrives to the depot, nearest to the last visited customer.

In table (2), the results obtained from a state that the primary and the last depots are equal, are compared with when they are different and also the percent of their answer's difference is showed. In table (3), results are compared with the result obtained by [4] and the proposed method gets 11 best solutions.

**Table 2.** A comparison of the answers obtained from the states of returning to the primary depot and not returning

| The kind of problem | The number of customers | The number of vehicles | The number of depots | The answer obtained from state that not return to the primary depot | The answer obtained from state that return to the primary depot | Gap (%) |
|---|---|---|---|---|---|---|
| P01 | 48 | 2 | 4 | 1052 | 1088 | 0.33 |
| P02 | 96 | 3 | 4 | 2401.7 | 2493 | 3.66 |
| P03 | 144 | 4 | 4 | 2352.4 | 2645.7 | 6.13 |
| P04 | 192 | 5 | 4 | 2501.4 | 2678.8 | 6.62 |
| P05 | 240 | 4 | 6 | 2292.8 | 2320.6 | 1.23 |
| P06 | 288 | 2 | 7 | 3011.3 | 3089.5 | 2.53 |
| P08 | 144 | 3 | 6 | 3146.8 | 3152.4 | 0.17 |
| P09 | 216 | 4 | 6 | 298806 | 3241.5 | 7.80 |
| P10 | 288 | 5 | 6 | 2695.6 | 2760 | 2.33 |
| P12 | 96 | 2 | 4 | 2924.6 | 3196.7 | 8.51 |
| P13 | 144 | 3 | 4 | 3682.9 | 3692.4 | 0.25 |
| P14 | 192 | 4 | 4 | 2802 | 2992.1 | 6.35 |
| P15 | 240 | 5 | 4 | 2494.5 | 2678.9 | 6.88 |
| P16 | 288 | 6 | 4 | 2392.4 | 2487.3 | 3.81 |
| P18 | 144 | 2 | 6 | 3623.8 | 3637.1 | 0.30 |
| P19 | 216 | 3 | 6 | 3350.2 | 3483.8 | 3.83 |
| P20 | 288 | 4 | 6 | 2870.1 | 2937.8 | 2.30 |

**Table 3.** Comparison to the solution reported in the literature.

| The kind of problem | The number of customers | vehicles | depots | not return to the primary depot | return to the primary depot | Cordeau et al [4] |
|---|---|---|---|---|---|---|
| P01 | 48 | 2 | 4 | 1052 | 1088 | 1074.12 |
| P03 | 144 | 4 | 4 | 2352.4 | 2645.7 | 2373.65 |
| P04 | 192 | 5 | 4 | 2501.4 | 2678.8 | 2819.76 |
| P05 | 240 | 4 | 6 | 2292.8 | 2320.6 | 2971.90 |
| P06 | 288 | 2 | 6 | 3011.3 | 3089.5 | 3590.58 |
| P08 | 144 | 3 | 6 | 3146.8 | 3152.4 | 2096.73 |
| P09 | 216 | 4 | 6 | 298806 | 3241.5 | 2717.90 |
| P10 | 288 | 5 | 6 | 2695.6 | 2760 | 3469.29 |
| P12 | 96 | 2 | 4 | 2924.6 | 3196.7 | 2925 |
| P13 | 144 | 3 | 4 | 3682.9 | 3692.4 | 3701.81 |

**Table 3.** Continued

| The kind of problem | The number of customers | vehicles | depots | not return to the primary depot | return to the primary depot | Cordeau et al [4] |
|---|---|---|---|---|---|---|
| P14 | 192 | 4 | 4 | 2802 | 2992.1 | 2895.33 |
| P15 | 240 | 5 | 4 | 2494.5 | 2678.9 | 2434.94 |
| P16 | 288 | 6 | 4 | 2392.4 | 2487.3 | 2852.25 |
| P18 | 144 | 2 | 6 | 3623.8 | 3637.1 | 2788.18 |
| P19 | 216 | 3 | 6 | 3350.2 | 3483.8 | 2263.74 |
| P20 | 288 | 4 | 6 | 2870.1 | 2937.8 | 2995.08 |

## 5. Conclusions

In this paper, we have studied one of the practical discussions in a real world. This subject is the multi-depot vehicle routing problem with time windows (MDVRPTW) which is one of the main problems in the field of logistics and transportation. Here, the new formulation of this problem is proposed in which the constraint of returning to the first depot is omitted and we considered a state in which the first and the last depots of the route can be different. Then, the offered algorithm has been applied for problem solving. The proposed algorithm is the hybrid of the clustering algorithms and the genetic heuristic algorithm in which the customers are divided into groups firstly to reduce search space and then, we have tried to find the shortest route among the customers by the genetic algorithm. We have presented the advantages of proposed methods by means of comparing it to two other clustering algorithms, K-means and FCM. The results obtained from each method have proved our idea and the effect of GA-clustering algorithm on minimizing the objective function of MDVRPTW problem. Obtained results were compared to the state that the primary and the last depots are equal, the percent of their answer's difference showed that formulation of this problem is better than previous formulations. furthermore, result are compared with the solution reported in the literature, and our results have indicated to better solutions in some cases. Finally, this study shows that our method can be applied successfully in the large water project problems.

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